Where on Earth are We?
Using the Sky for Mapping the Nordic Countries 1500 – 2000

Martin Ekman

Summer Institute for Historical Geophysics
Åland Islands

Where on Earth are We?
Using the Sky for Mapping
the Nordic Countries 1500 – 2000

Martin Ekman

Summer Institute for Historical Geophysics
Åland Islands

Summer Institute for Historical Geophysics, Åland Islands

The Summer Institute for Historical Geophysics is the author's one-man-institute, performing research, issuing publications and now and then giving lectures within the field of historical geophysics. For further information, or for access to publications in the series "Small Publications in Historical Geophysics" issued by the institute, go to:

www.historicalgeophysics.ax

ISBN 978-952-92-9137-3

Printing original by Ben Johans, PQR

Printed by Logotipas

Published by the Summer Institute for Historical Geophysics, Äppelträdgården E, Haraldsby, AX – 22 410 Godby, Åland Islands. See also above.

Cover picture: A starlit sky during a Nordic winter night. Painting by Harald Wiberg 1960 as an illustration to Viktor Rydbergs poem "Tomten" from 1881.

Contents

Preface

This is a scientific book spanning from astronomy via geodesy to mapping (and partly navigation). Its character is somewhat similar to that of the author's earlier book "The changing level of the Baltic Sea during 300 years: A clue to understanding the Earth". Yet the present book has its own flavour.

First, the theme of the book may be described as: How to use the sky to find positions on the Earth in order to construct a map. What we study here is the fundamental positioning for the mapping of the Nordic countries during five centuries. This is of a wider international interest than might be assumed, as will be shown.

Second, the book has a combined scientific and historical perspective throughout. On one hand, science is used to contribute to understanding the historical development of positioning and mapping. On the other hand, the historical development is used to contribute to understanding the principles behind modern scientific positioning. Original scientific sources (and maps) are used throughout. This means throwing light also on important works no longer known to modern scientists.

Third, in order to broaden the outlook in somewhat unexpected directions, some special aspects related to the positioning and mapping problems are included at the end of the book.

The book is intended for reading by a wide range of geoscientists or other people with a professional interest in the mapping of the Earth. The reader is assumed to have an elementary knowledge of basic Earth science, but not to know anything about positioning. The reading of the book will be facilitated by understanding nature's own language, mathematics, at the level of a novice at a university. In some cases more advanced mathematical concepts occur in the text; these can simply be passed over by those who are not familiar with them. The book might also be of interest for historians of science, but they should be aware that this is a book about science rather than about history of science.

There are a number of quotations in the text, serving to illustrate and bring to life important points in the scientific development. Their translations into English are due to the present author. Some of the older quotations have had to be translated rather freely to make them readable; however, great care has always been taken to convey the original message in a correct way.

References in the text are given by names and years within brackets; when years occur without brackets they relate to historical information. In order to make the scientists and other persons referred to less anonymous, they have all been given a brief characterization, such as German astronomer and geodesist, Danish-Swedish cartographer etc. The reference list at the end is not ordered alphabetically, as would be normal, but chronologically. This has made it possible to give a chronological overview of all the published works used.

The typographical layout of the book, including the design of its cover, is due to the author. This was the case also with the earlier book; the principles of its layout have been kept in the present one.

Most of the author's work on this book has been performed within his private one-man-institute, the Summer Institute for Historical Geophysics, on the Åland Islands in the Baltic Sea. Some early inspiration stems from the time when the author was working at the Geodetic Research Division of the National Land Survey of Sweden.

A number of people have been helpful during the work with this book and the research leading up to it. I would like to thank the persons who have read the manuscript and given constructive comments on it: Jonas Ågren at the National Land Survey of Sweden, Niels Andersen at the Danish Space Centre, Bjørn Geirr Harsson at the Norwegian Mapping Authority, and Jaakko Mäkinen at the Finnish Geodetic Institute. To find the old literature I have had great benefit of the library of the Royal Swedish Academy of Sciences, partly deposited with the University Library of Stockholm, and the library of the former Geographical Survey Office of Sweden, to some extent also the library of the Astronomical Observatory at the University of Uppsala and the library of the former Danish Geodetic Institute. To find some original positioning information not published I had great use of the Geodetic Archives of the National Land Survey of Sweden as well as the Archives of the National Maritime Administration of Sweden.

My final thanks go to those star observers, long since dead, whose faithful work in former centuries during cold and dark nights created a foundation for the mapping of the Earth's surface and, thereby, for this book.

Åland Islands, a starlit evening in autumn 2010

Martin Ekman

1. Introduction: Combining the sky and the Earth

1.1 How does one make a map?

This is a book about 500 years of Nordic answers to a very specific question: Where on Earth are we? The easiest way to answer such a question is to look on a map of the Earth, find your location on the map, and say, pointing at it: Here we are! But how can this question be answered when we do not have a map – if we, instead, are the ones who have been charged with making the map? Then we will have to find our position on the Earth in some other way, by turning our eyes towards the sky, as in Figure 1-1. We will have to observe sun, stars, moons, satellites etc. This is what this book is about. The book deals with how the sky has been used to map the Earth in the north, from the 1500s up till today. Furthermore, the book investigates how the uncertainty of the positions has decreased during these five centuries, from, as it will turn out, one hundred kilometres to a centimetre.

The construction of a map or a chart from the very beginning is a task that has to be solved in several steps as follows:

1. Find the *size and shape of the Earth*. In the early days the Earth was treated as a sphere, characterized by its radius. Later on it was realized that the Earth is flattened at the poles due to its rotation around its axis and thus has to be treated as an ellipsoid, characterized by its equatorial radius and its flattening.

2. Determine the *positions of a number of points on the Earth's surface* as accurately as possible. The position is given by two coordinates, latitude and longitude. Other points are then determined locally in relation to these fundamental points. This item, *positioning*, forms the main part of making a map; it is a time-consuming work. Calculations are made on the spherical or ellipsoidal Earth defined by item 1.

3. Perform a *projection of the curved surface of the Earth onto a plane*, a *map projection*. This means mathematically converting all latitudes and longitudes, obtained according to item 2, to planar coordinates, in such a way

Figure 1-1. A starlit sky during a Nordic winter night. (Painting by H Wiberg 1960 as an illustration to V Rydbergs poem “Tomten” from 1881.)

that inevitable distortions of the Earth’s surface caused by the projection are in some sense minimized.

4. Now the *map is ready to be drawn,* and then printed on a sheet of paper. The final product is, hopefully, a combination of science and art.

1.2 Northern positioning with international connections

There are certain things that make positioning and mapping of the Nordic area through history of a wider international interest than might be assumed. These things are mainly related to scientific activities at special observatories and to scientific expeditions requiring high latitudes.

The observatory of *Uranienborg,* close to København, was founded in 1576 by the Danish astronomer Tycho Brahe, known for his careful observations leading to Kepler’s laws of planetary motion, and from there on to Newton’s law of gravitation. Uranienborg at that time developed into a kind of international centre for positioning of stars on the celestial sphere as well as for positioning of places on the Earth, especially for determinations of latitude.

The observatory of *Uppsala,* not far from Stockholm, was founded in 1739 by the Swedish geophysicist Anders Celsius, mostly known for his temperature scale. A few decades later much of the activities there were moved to the new observatory of Stockholm. Through Celsius' assistant, Pehr Wargentin, Uppsala and later Stockholm for a period became an international centre for determinations of longitude using the moons of Jupiter.

The fact that the Nordic countries form a fairly inhabited area more to the north than anywhere else in the world has made this area an interesting one from a geoscientific point of view. Already in the middle of the 1700s there was a French initiative for determining the distances to the moon and the sun by making observations from locations on the same longitude but maximally separated in latitude. The optimal areas were found to be South Africa and northern Sweden; this contributed to promoting astronomical positioning in Sweden at that time.

In the beginning of the 1700s French scientists tried to solve an international controversy on the flattening of the Earth and, ultimately, on Newton's theories of gravitation and rotation. They therefore sent a scientific expedition, including Celsius, as far north as possible, which was judged to be northern Sweden with Finland. This had a considerable impact on the introduction of triangulation for mapping purposes in the Nordic countries, especially for charting the Baltic Sea. A century later, in the middle of the 1800s, there was a Russian expedition, with Scandinavian participants, for investigating the flattening of the Earth; this had impacts on the mapping of Finland and northern Norway. And at about the same time Gauss in Germany, applying his new mathematics to mapping problems, cooperated with the mapping scientists in Denmark.

In our own time the introduction of satellite positioning has been promoted by the existence of a radio-astronomical observatory at Onsala close to Göteborg and a Nordic-American cooperation connected to that.

It should be clarified here that the book deals with the main parts of the Nordic countries Denmark, Norway, Sweden and Finland, including the Baltic Sea. It does not include the Nordic islands in the North Atlantic, except when these have been used for the introduction of new methods in the Nordic area.

1.3 Coordinates on the Earth and in the sky

Before entering into the main contents of the book we need to define the coordinates we are going to use, both on the Earth and on the celestial sphere.

A position on the Earth is specified by its latitude and longitude. On a *spherical Earth* these well-known coordinates are simple to define; see Figure 1-2. The *latitude* is the arc along a meridian from the equator to the point in question, or the corresponding angle between the equatorial plane and the normal to the sphere through the point. The *longitude* is the arc along the equator between an arbitrary zero meridian and the meridian through the point in question, or the corresponding angle between the two meridian planes.

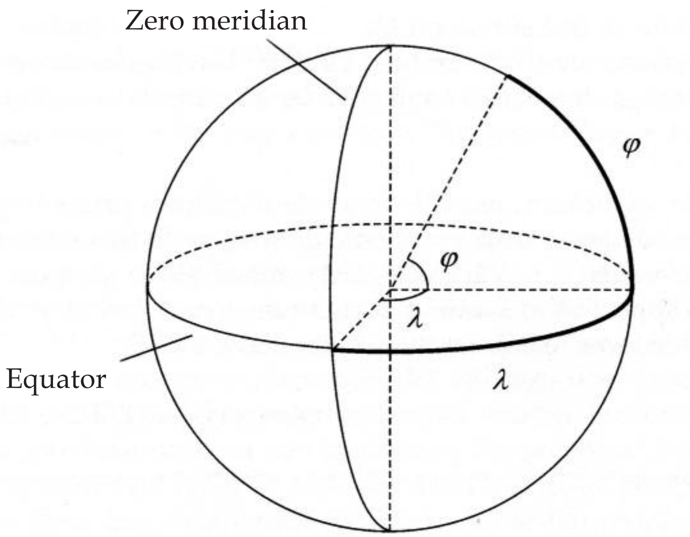

Figure 1-2. Latitude (φ) and longitude (λ) on a spherical Earth.

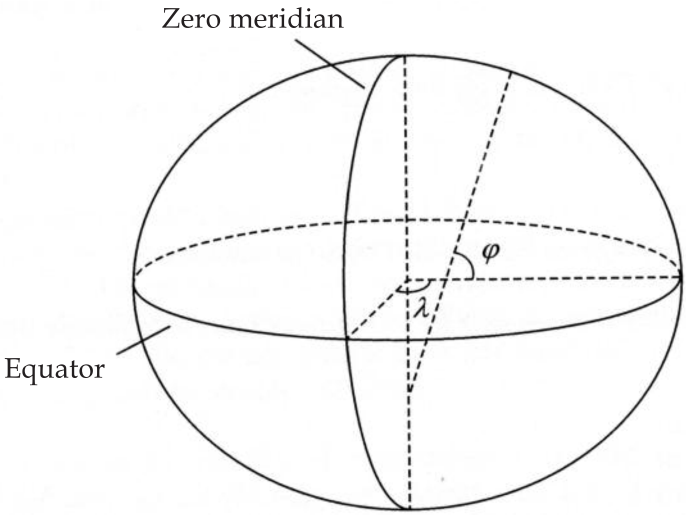

Figure 1-3. Latitude (φ) and longitude (λ) on an ellipsoidal Earth flattened at the poles.

On an *ellipsoidal Earth* flattened at the poles things become a little more complicated; see Figure 1-3. The *latitude* can no longer be defined as an arc since the meridian now is elliptic. The latitude has to be defined as the angle between the equatorial plane and the normal to the ellipsoid through the point in question. The *longitude* can still be defined as above. (Later in the book it will turn out that disturbing influences of the irregular mass distribution within the Earth will make things even more complicated, but we leave that aside here).

Turning to the sky we can more or less copy the spherical coordinate system from the Earth to the *celestial sphere,* with the Earth in its centre. Thus we have on the celestial sphere a celestial equator. The coordinate here corresponding to the latitude is known as the *declination.* The coordinate corresponding to the longitude is known as the *right ascension;* it is counted from a "zero meridian" through the so-called vernal equinoctial point (this is where the sun is at the vernal equinox).

None of the coordinates above can be observed directly. What can be observed are only the coordinates of sun, stars etc. with respect to the horizon. These coordinates are the *altitude,* reckoned from the horizon upwards, and the *azimuth*, reckoned from the meridian in the south towards west.

Once the declination and right ascension of a celestial object are known, observations of its altitude and azimuth can yield the latitude and longitude of the observation point on the Earth. If, on the other hand, the latitude and longitude of the observation point are known, observations of altitude and azimuth can yield the declination and right ascension of the object. But if the latitude and longitude are not known, the declination and right ascension cannot be found, and if the declination and right ascension are not known, the latitude and longitude cannot be found. So where does it all start? How can we find a position on the Earth without knowing anything beforehand?

A modern person would answer: Use satellites! But satellite positioning is based on measuring distances from the satellites to the observation point. To find the coordinates of the observation point from the distances we need to know the coordinates of the satellites. To find the coordinates of the satellites we have to observe them from some observatories on the Earth, and then we need to know the coordinates of these observatories. But how then do we find the coordinates of the observatories? This only leads us back to the original question: Where does it all start? How can we find a position on the Earth without knowing anything beforehand? That will be revealed in the following chapters.

2. Sun and latitude: General maps of the first generation

2.1 The sun at the Arctic circle

The simplest way of finding one's *latitude* on the Earth is by observing the *sun*, more specifically its altitude (height angle) above the horizon. This principle had been known already to the Greeks during the antiquity. Eratosthenes had used it, together with distance measurements along a meridian, to find the radius of the Earth about 240 B.C.

Observing the altitude of the sun was probably the major tool for navigation used by the Norwegian and Icelandic Vikings, who were the first to cross the North Atlantic Ocean one thousand years ago. The use of the sun for finding the latitude here was facilitated by the sun being above the horizon for most of the twenty-four hours during summer.

The oldest preserved document related to the need for determining the latitude when sailing on the Atlantic is a manuscript by the Icelandic navigator Oddi Helgason (c. 1150). Helgason, known as Stjörnu-Oddi [Star-Oddi], made a compilation of the daily maximum altitude of the sun above the horizon during the course of a year. This was based on his observations of the sun performed on a small and flat island, Flatey, situated in a bay on the northern coast of Iceland, very close to the Arctic circle. Let us study the main results of Stjörnu-Oddi in order to understand how they might be used for latitude determinations.

Stjörnu-Oddi measured the sun's altitude above the horizon at noon, when the sun is in the meridian, i.e. due south. This is when the sun has its maximum altitude during the day and, consequently, its shadow is at its shortest. The angular unit used by him for giving the sun's altitude is a ”wheel”, one wheel being equal to the angular diameter of the sun. Hence 1 wheel = 32′ = 0.53°.

During a year the sun is at its lowest at the winter solstice (normally 22 December). On that occasion the sun could not be seen from Flatey because of the mountains on the mainland in the south. However, Stjörnu-

Oddi must have been able to estimate that the meridian altitude of the sun was close to zero that day, because the same altitude could be observed for the midsummer midnight sun at the free horizon in the north. Starting from the winter solstice Stjörnu-Oddi then gives an account of how the meridian altitude of the sun increases with the time of the year, until it reaches its highest value at the summer solstice half a year later (normally 22 June). He finds that the total increase in meridian altitude amounts to 91 wheels. After this the altitude decreases in the same way during half a year until the next winter solstice. Denoting the meridian altitude of the sun at winter solstice by h_w, at summer solstice by h_s, and at the equinoxes in between by h_e, we have according to Stjörnu-Oddi h_w = 0 wheels = 0°, h_e = 45 ½ wheels = 24°, h_s = 91 wheels = 48°. The correct values would be, respectively, 0°, 23 ½ °, 47° (the effect of refraction close to the horizon at the winter solstice being ignored).

At the vernal equinox (normally 21 March) or the autumnal equinox (normally 23 September), when day and night are of equal length, the sun is in the equator. This means that the equator can be easily located on the celestial sphere on that occasion; it is simply where the sun is. As a consequence, the latitude φ of the observation point on the Earth can be determined directly from the meridian altitude of the sun above the horizon that day:

$$\varphi = 90° - h_e \tag{2-1}$$

Inserting h_e = 24° from Stjörnu-Oddi's data above we find φ = 66°. The correct value for the Arctic circle is 66 ½ °, making the error in latitude only ½ °.

Once the latitude is fixed, the sun's deviation from the equator, i.e. the declination δ of the sun, can be determined throughout the year: $\delta = h - h_e$ or

$$\delta = h + \varphi - 90° \tag{2-2}$$

Here h denotes the meridian altitude of the sun each day of observation. From Stjörnu-Oddi's data we find, with the correct values within brackets, δ_w = - 24° (- 23 ½ °), δ_e = 0° (0°), δ_s = + 24° (+ 23 ½ °).

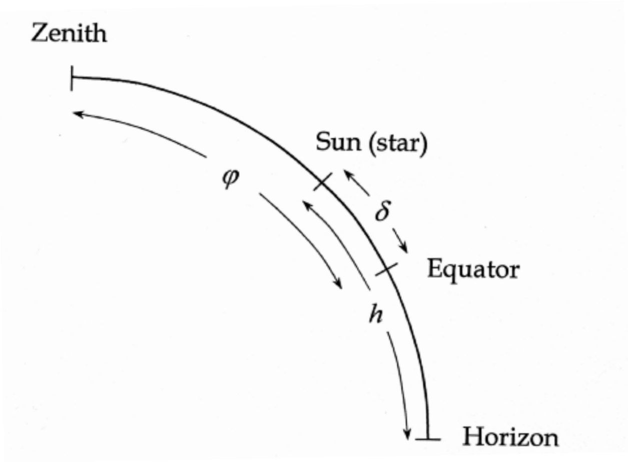

Figure 2-1. The relation between latitude (φ), declination (δ) and altitude (h) along the meridian on the celestial sphere.

With such a table of the declination of the sun, in a more detailed version, the latitude of any point on the Earth can be determined any day of the year by measuring the meridian altitude of the sun as seen from the point in question:

$$\varphi = \delta + 90° - h \tag{2-3}$$

In this way Stjörnu-Oddi's data could be used for latitude determinations, e.g. during navigation on the ocean; see also Figure 2-1.

Looking back we may describe the whole procedure of *latitude determination, using observations of the sun,* in the following three steps:

1. Determine the latitude of an "observatory" from the meridian altitude of the sun there at the vernal or autumnal equinox, according to (2-1).

2. Determine the declinations of the sun throughout the year from the latitude found in item 1 and the meridian altitudes of the sun at the "observatory" throughout the year, according to (2-2).

3. Determine the latitude of an arbitrary station from the declinations of the sun found in item 2 and the meridian altitude of the sun at the station in question any day of the year, according to (2-3).

2.2 The first maps based on sun observations

In the 1500s ships started crossing the oceans on a more global scale. By collecting sun-based latitude determinations and other data from ships navigating the seas and visiting harbours along the coasts, it became possible to make reasonably realistic maps of the world. These maps included a graticule, i.e. parallels and meridians, to allow reading of latitudes and longitudes for various objects on the map. The first useful world map of this kind was constructed by the Dutch mathematician and cartographer Gerard Mercator (1569). Mercator had produced a smaller such map already in 1538, but this was a large one. Moreover, with this map Mercator introduced a new map projection; later on this projection was to be used for marine charts all over the world.

Mercator's map was issued only in a few copies. However, a collaboration with another Dutch cartographer, Abraham Ortelius, resulted in a large number of more detailed maps. They were all produced in equal size and put together into a book. This was the first atlas; it was issued by Ortelius the year after Mercator's map, Mercator acting in the background as a kind of scientific advisor. The maps of the atlas were drawn and engraved on copper plates, and then printed and coloured. The colouring of the maps led to one of the world's first salaried employments for women, among them Ortelius' sisters.

In the atlas one finds what is probably the *oldest astronomically based map of the Nordic countries and the Baltic Sea.* This map of Ortelius (1570) is the oldest Nordic map with a realistic graticule of parallels and meridians appearing on it; see Figure 2-2. It should be mentioned here that Mercator had produced a similar map earlier, in 1554, but no copy of that map has survived until today. Both Ortelius and Mercator included in their maps information from more primitive maps issued earlier, especially that of the exiled Swedish cleric Olaus Magnus, but the scientific basis of their maps is completely new.

Let us now use the graticule on Ortelius' map to investigate the latitude determinations behind it, most of them apparently compiled by Mercator from about 1538 onwards. Stations selected for the investigation are harbour towns and alike at that time; for such places latitudes might have been reported from ships navigating the Nordic and Baltic

Figure 2-2. Map of the Nordic countries and the Baltic Sea (and a part of the North Atlantic) by Ortelius (1570), the oldest map based on latitudes from sun observations (compiled by Mercator).

area. The 26 selected stations are listed in Table 2-1, together with their latitudes read from the map, their latitudes according to modern knowledge (correct latitudes), and the error in latitude for each station. The error is calculated as map latitude minus modern latitude. The map latitudes have been read with an accuracy of 5′; accordingly they as well as the latitude errors are given in 5′ intervals.

A group of stations at the top of the table, marked by asterisks, have very large errors, about 3°. This indicates that their latitudes cannot have been determined astronomically. They are situated in the innermost parts of the Gulf of Finland and the Gulf of Bothnia, respectively, all of them being at that time remote and small places with the sea being ice-covered during two thirds of the year. Two stations at the bottom of the table, also marked by asterisks, have for unknown reasons also unrealistic

Table 2-1. Latitudes (in degrees and minutes) on the map of Ortelius (1570) and their errors. Stations are in anti-clockwise order along the coasts of the Baltic Sea and adjacent waters. For explanations of asterisks see text.

Station	Map	Modern	Error
Viborg	64° 30′	60° 43′	3° 45′ *
Åbo (Turku)	61 15	60 28	45
Korsholm [Vasa]	65 40	63 04	2 35 *
Torneå (Tornio)	68 50	65 51	3 00 *
Umeå	65 25	63 50	1 35 *
Gävle	61 30	60 40	50
Uppsala	60 25	59 52	35
Stockholm	60 10	59 20	50
Visby	57 25	57 39	- 15
Kalmar	57 10	56 40	30
Älvsborg [Göteborg]	59 15	57 43	1 30
Oslo	60 25	59 55	30
Stavanger	60 05	58 58	1 05
Bergen	60 45	60 24	20
Trondheim	64 45	63 26	1 20
Tromsø	70 45	69 39	1 05
Vardø	70 50	70 22	30
Århus	56 45	56 10	35
København	56 35	55 41	55
Lübeck	54 15	53 52	25
Stralsund	54 00	54 18	- 20
Danzig (Gdansk)	53 45	54 21	- 35
Königsberg (Kaliningrad)	54 30	54 42	- 10
Memel (Klaipeda)	55 10	55 43	- 35
Riga	59 00	56 57	2 05 *
Reval (Tallinn)	61 25	59 26	2 00 *

errors. The stations thus questioned have been excluded from the further analysis.

Analysing the remaining 20 stations in Table 2-1 we find two things. First, their errors in latitude have an average indicating a systematic error in latitude of

$$\Delta\varphi = 30'$$

This could be due to some defect in the altitude measurements of the sun or to some defect in the declinational tables of the sun, or both. A possibility is that the observations have not always been made precisely in the meridian; such an inaccuracy would make the altitudes too small and the latitudes correspondingly too large (see Section 2.1). Second, subtracting these 30′ the remaining errors yield a standard deviation in latitude of

$$\sigma_{\varphi} = 35'$$

This is thus the effect of random errors. Some slight regional tendencies may, however, be noticed. Such effects might be due to the latitudes within a group of stations having been reported from one and the same ship, or to only one of these latitudes having been measured astronomically and the other ones having been estimated in relation to the measured one. The general uncertainty in latitude found here corresponds to about 100 km on the Earth's surface (1° ≈ 111 km).

On the whole, the impression is that the accuracy in latitude has not improved considerably since the days of the Vikings. What is new is that latitude data have been actively collected from a number of harbours and thereby have made it possible to start mapping the Earth.

Finally, what about the longitudes on the map? Well, longitudes at that time could not be measured directly. They had to be crudely estimated with indirect methods, typically from recorded courses and estimated distances. Longitudes did not begin to be measured until a century later; they will appear in Chapter 4.

3. Stars and latitude: General maps of the second generation

3.1 The observatory on the island: The stars at Uranienborg

In 1576 the Danish astronomer Tycho Brahe moved to the small and flat island of Ven (at that time spelt Hven), situated in the entrance to the Baltic Sea. The King had given him the right to use this island for erecting an astronomical observatory. The building that Brahe erected was a remarkable combination of an astronomical observatory and a decorated palace, surrounded by a geometrical garden; see Figure 3-1. He named this establishment *Uranienborg* (after Urania, the protectress of astronomy in Greek mythology). Later a group of small separate observatory buildings was added.

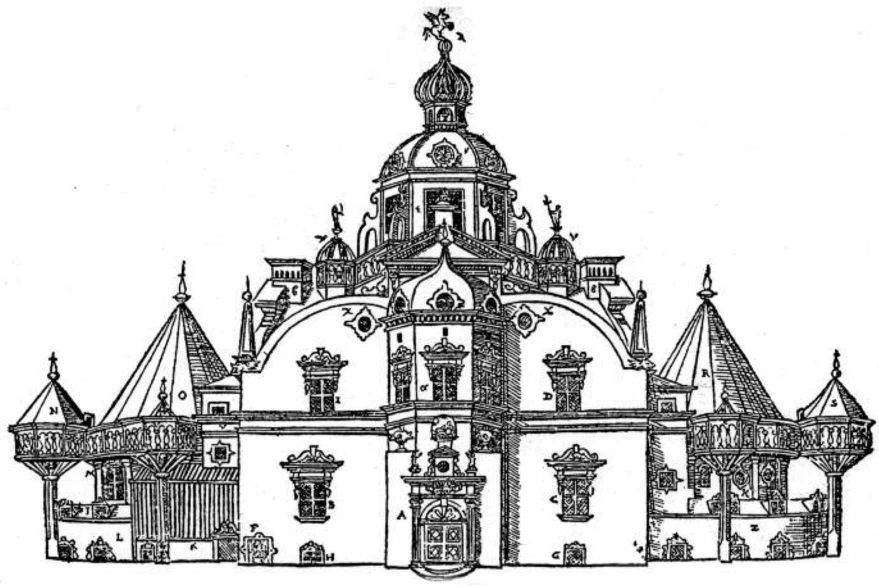

Figure 3-1. The observatory of Uranienborg according to Brahe (1596). Today only small ruins remain.

Figure 3-2. The large quadrant in Uranienborg, used for determining the latitude of the observatory from the altitudes of stars. The persons in the foreground are observers; the area above the quadrant itself is covered by a wall painting. (Brahe, 1598.)

Brahe constructed his own instruments, designing them to give an accuracy never achieved before. Moreover, he checked his various instruments against each other. His main instrument was a large quadrant for measuring vertical angles in the meridian, mounted on a stable and specially painted wall in the main building; see Figure 3-2. With this instrument and others he could accurately measure the altitude of stars and other celestial objects above the horizon; Brahe (1598) claims that the altitudes could be read within 1/6 of a minute (10″). Observations were performed almost every clear night during a period of 20 years – just with the naked eye, as the telescope had not yet been invented. The observations required a number of research assistants, educated by

Brahe, one of them being his favourite sister, Sophie Brahe. She must have been one of the first female scientists in the world.

The first thing to do at the observatory of Uranienborg was to determine its *latitude* through observing a *star;* this becomes more accurate than observing the sun. In Chapter 2 the meridian altitude of the sun at the equinoxes was used for this purpose through identifying the equator on the celestial sphere. The stars cannot be used in the same way. The trick here is to use a star close to the pole to identify the pole on the celestial sphere. (There is no star in the pole itself, not even the pole star.)

A star close enough to the pole will be above the horizon all the time and move in a circle around the pole during a day, reflecting the Earth's rotation. Such a circumpolar star will transit the meridian twice a day, once on the upper side of the pole, upper culmination, and once on the lower side, lower culmination. Let us denote the altitude of the star above the horizon at upper culmination by h_u and at lower culmination by h_l. Then the altitude of the pole above the horizon can be identified as the mean value of the two observed altitudes of the star, and this is equal to the latitude φ of the observation point:

$$\varphi = \frac{h_u + h_l}{2} \tag{3-1}$$

Formula (3-1) for the circumpolar star should be compared with formula (2-1) for the sun. In (2-1) the latitude is determined from the *altitude of the equator above the horizon,* in (3-1) the latitude is determined from the *altitude of the pole above the horizon.*

Tycho Brahe measured the two meridian altitudes of the pole star as well as other circumpolar stars to determine the latitude of Uranienborg. He continued with this work through all the 20 years he spent there. Finally Brahe (1596) published a value of the latitude, φ = 55° 54 ½'. This value is not stated in the text but on a small map of Ven with Uranienborg shown at the mentioned latitude, given on the map as 55°54'30". In his next work, Brahe (1598) publishes the same map again, now stating explicitly:

"The island itself is very high, as if it were a mountain which you might ascend, but on top it is flat all over. In the centre, where I have built the palace of Uranienborg, the altitude of the pole or, what amounts to the same, the latitude from the equator is 55° 54 ½′, as measured several times by us with the greatest care."

The map was later republished in an amended version by one of his assistants, the Dutch astronomer and cartographer Willem Blaeu and his son Joan Blaeu (1638), in a famous atlas of theirs.

Brahe's latitude determination was of an unprecedented accuracy. The modern (correct) value is actually 55° 54 ½′ – the same as Brahe's result. This does not mean, however, that there was no error at all in his determination. Correcting his observed altitudes and, thereby, his latitude for refraction (see below), we obtain φ = 55°54′, still yielding an error in latitude as small as ½′! A summary of this and a few later latitude determinations, to be discussed in Section 3.4, are given in Table 3-1.

With the latitude now fixed Brahe could determine the declination of any star through

$$\delta = h + \varphi - 90° \qquad (3\text{-}2)$$

h being the meridian altitude of the star. This is the same formula as (2-2) for the sun. After having measured the meridian altitudes of a large

Table 3-1. Latitude determinations (in degrees and minutes) of Uranienborg in the 1500s and 1600s, and their errors. Correction for refraction applied here where necessary.

Data source	Measured	Corrected	Modern	Error
Brahe (1596)	55° 54.5′	55° 53.8′	55° 54.4′	- 0.6′
Brahe (unpubl.)	55 54.7	55 54.0	55 54.4	- 0.4
Picard (1680), pole	55 55.3	55 54.7	55 54.4	0.3
Picard (1680), zenith	55 54.2	55 54.2	55 54.4	- 0.2

number of stars, Brahe (1602) calculated and published their declinations in a star catalogue. The accuracy of the declinations is of the order a minute of arc. This was a great achievement; the catalogue was later included by the German astronomer Johannes Kepler (1627) in his astronomical tables and became an international standard for nearly a century.

With the declinations in Brahe's star catalogue the latitude of any point on the Earth could be determined by measuring the meridian altitude of a star as seen from the point in question,

$$\varphi = \delta + 90° - h \tag{3-3}$$

This formula is identical to (2-3) for the sun.

When applying (3-3) one has to be aware of a special phenomenon affecting the result. This phenomenon is an apparent increase in the observed altitude of a star caused by refraction of the star light in the atmosphere. For this effect Brahe worked out a table showing the approximate refraction as a function of the observed altitude. He found that refraction was insignificant for altitudes above 45° (not quite correct, hence his lack of refraction correction in his latitude for Uranienborg), and increased with decreasing altitudes to reach a maximum of 30′ – 35′ at the altitude of 0°, i.e. at the horizon. Accordingly it was important to use stars fairly close to zenith for accurate observations.

On the whole, Brahe's work opened up completely new possibilities to make latitude determinations for accurate mapping. Looking back we may describe the whole procedure of *latitude determination, using observations of stars*, in the following three steps:

1. Determine the latitude of an observatory from the mean of the two meridian altitudes of a circumpolar star there, according to (3-1).

2. Determine the declinations of stars from the latitude found in item 1 and the meridian altitudes of the stars at the observatory, according to (3-2).

3. Determine the latitude of an arbitrary station from the declination of a star as found in item 2 and the meridian altitude of the star at the station in question, according to (3-3).

3.2 The first maps based on star observations

On Christmas Eve in 1602 the Swedish astronomer and historian Johan Bure was summoned before the King. What was the important matter to be dealt with such a day? The King wanted Bure to give a report on a determination of the latitude of a tiny place in the north! The tiny place was Torneå, at the northern end of the Gulf of Bothnia, in what today is Finland.

Apparently Bure's report on the latitude determination was satisfactory. The year after, in 1603, the King ordered Bure's cousin, the mathematician and cartographer Anders Bure, to prepare a large map of the Nordic countries. The two cousins worked together, Johan Bure mostly with building astronomical instruments and determining latitudes, Anders Bure mostly with collecting other data and drawing the map. A preliminary map of the northernmost parts, around the Arctic circle, appeared first. Then, after more than twenty years, Anders Bure (1626) could present the final *map over the whole Nordic area,* the *first one based also on star observations.* It was an impressive map, printed only in a few copies. Soon, however, it was republished in a smaller version in, among others, the famous atlas of Blaeu and, thereby, widely spread. The map is shown in Figure 3-3.

Let us now use the graticule on Bure's original map to investigate the latitude determinations behind it, most of them probably performed by Johan Bure or according to his instructions. The stations selected for the investigation are the same as those used for the Mercator-Ortelius map in Chapter 2, thereby facilitating a comparison between the two maps. The 26 stations are listed in Table 3-2 together with their latitudes read from the map, their latitudes according to modern knowledge (correct latitudes), and the error in latitude for each station. The map latitudes have been read with an accuracy of 1′. The error is calculated as map latitude minus modern latitude.

We first note that a group of stations in the middle of the table, marked by asterisks, have large errors, up to about 30′. These are all Norwegian and Danish stations (including one under temporary Danish control). This obviously has to do with the political situation in the be-

Figure 3-3. Map of the Nordic countries and the Baltic Sea by Bure (1626), the first map based on latitudes from star observations. The map shown here is the atlas version of Blaeu.

ginning of the 1600s. Sweden, at that time including Finland and Estonia, and Denmark, at that time including Norway in a kind of union, were in conflict with each other. The Swedes, therefore, could measure latitudes in more or less the whole area around the Baltic Sea, but not in Denmark and southern Norway. There Bure apparently had to rely mostly on already published maps. Accordingly, these stations have been excluded from the further analysis. In the far north, however, north of the Arctic circle, there were no borders at that time; most of the people living there were Sami, with some Norwegians at the Arctic Sea. In this area the Swedes, as far as can be judged, measured latitudes right up to the Arctic Sea, hence one station without an asterisk there.

Table 3-2. Latitudes (in degrees and minutes) on the map of Bure (1626) and their errors. Stations are in anti-clockwise order along the coasts of the Baltic Sea and adjacent waters. For explanations of asterisks see text.

Station	Map	Modern	Error
Viborg	60° 50′	60° 43′	7′
Åbo (Turku)	60 29	60 28	1
Korsholm [Vasa]	63 08	63 04	4
Torneå (Tornio)	65 54	65 51	3
Umeå	63 56	63 50	6
Gävle	60 35	60 40	- 5
Uppsala	59 43	59 52	- 9
Stockholm	59 14	59 20	- 6
Visby	57 39	57 39	0
Kalmar	56 44	56 40	4
Älvsborg [Göteborg]	57 25	57 43	- 18 *
Oslo	59 25	59 55	- 30 *
Stavanger	58 38	58 58	- 20 *
Bergen	60 10	60 24	- 14 *
Trondheim	64 06	63 26	40 *
Tromsø	69 34	69 39	- 5
Vardø	70 37	70 22	15 *
Århus	55 57	56 10	- 13 *
København	55 32	55 41	- 9 *
Lübeck	53 48	53 52	- 4
Stralsund	54 15	54 18	- 3
Danzig (Gdansk)	54 19	54 21	- 2
Königsberg (Kaliningrad)	54 40	54 42	- 2
Memel (Klaipeda)	55 47	55 43	- 4
Riga	56 52	56 57	- 5
Reval (Tallinn)	59 14	59 26	- 12

Analysing the remaining 17 stations in Table 3-2 we find two things. First, there seems to be no systematic error in latitude, only random errors. Second, the errors yield a standard deviation in latitude of

$$\sigma_{\varphi} = 5'$$

corresponding to an uncertainty of about 10 km on the Earth's surface. This is a remarkable improvement in comparison with the Mercator-Ortelius map in Chapter 2 (or other earlier maps). On the whole we may conclude that the accuracy of the latitude determinations behind the map of Bure has increased by one order of magnitude, i.e. by a factor 10 approximately. This should be associated with mostly observing stars rather than the sun, but much more than that is required.

The considerable increase in latitude accuracy has to depend on correspondingly increased accuracies in both altitudes and declinations of the observed stars (and sun). The altitudes of the stars were measured with new instruments, most probably copied from some of those Brahe had published in one of his books. The declinations of the stars must have been taken from the recent star catalogue of Brahe (1602). It is striking that Bure's report to the King on latitude determination, as well as the King's order to prepare the map, were given within one year after the appearance of Brahe's star catalogue.

We may conclude that the success of the Bure map of the Nordic area to a large extent rests on the works carried out by Brahe at Uranienborg. In particular, the map could hardly have been produced without his star catalogue, and the star catalogue could not have been produced without his excellent latitude determination at Uranienborg!

This having been said, we should not underestimate the work performed by Bure and his assistants. They had to travel long distances with their scientific instruments through forests and wildernesses. The observations of stars could not be performed during summer since the nights in the north are too bright. In winter, on the other hand, when it is dark enough for observing stars, and the snow allows travelling by sledge, it is often disturbingly cold. And from another positioning expedition made later in the same century (Bilberg, 1695) we have an account of typical travelling problems during spring, when it is no longer possible to transport the instruments by sledge:

”In the north we were hindered by snow and ice, and had several rivers and sounds to cross, some of them quite wide. Since there were mostly no ferries one was compelled to put horses and wagons into two boats tied together, two wheels in each boat. ... The frost in the ground had in some places broken up, making the wheels sink down; there we had to walk on foot for half a Swedish mile [5 km] or more. ... The bays of the sea were still covered by thick ice, and the spruce branches showing the winter road were still there, but since the ice no longer was connected to the land nobody dared to cross the bays.”

Under these conditions a standard deviation in latitude of 5′ must be considered a very successful result.

Now, what about Denmark and southern Norway, where Bure apparently had no access to recent latitude data? Well, here Brahe and a few of his research assistants had made latitude determinations as a basis for a future map, intended to cover Denmark and southernmost Norway. The map, however, never materialized and the results were not available to the outer world. Nevertheless, the latitude results are known from Brahe’s annotations; comparisons with modern latitudes indicate a standard deviation of just a few minutes. Later on the latitudes seem to have been partly used for the maps of Denmark and southern Norway in Blaeu’s atlas; Blaeu had, as mentioned earlier, also been one of Brahe’s assistants.

The latitude of Uranienborg is of particular interest in this connection. We have seen that this latitude had been published already by Brahe (1596), 55° 54 ½′. Still the centre of the island of Ven on Bure’s map is located on latitude 55°45′, which is 9′ to the south. The reason for this might be that Bure could not correct the position of Ven only, and he did not have information enough on how to modify the surrounding parts of the map. Blaeu, on the other hand, did have such information and thus could later on present more accurate maps of the Danish area; on his maps showing Denmark, Ven has been located according to the latitude of Brahe.

The main competence in positioning and mapping at this time was concentrated to Denmark and the Netherlands through Brahe and Blaeu. This is reflected also in the first Nordic text-book on accurate positioning, written by the Swedish mathematician and astronomer Bengt Hedraeus (1643). The book relies on Brahe concerning the use of stars, and is published in the Netherlands while the author was studying there.

Towards the end of the century the *first chart over the entire Baltic Sea* appeared. It was published by the Swedish naval officer Werner von Rosenfeldt and his colleague, the land surveyor Petter Gedda (1694), by order of the King. From the latitude point of view this chart shows almost no improvement since the map of Bure. On the other hand this is the first chart of the Baltic Sea constructed in the Mercator projection.

Finally we should mention that longitudes, as in Chapter 2, were still very difficult to measure. Thus also for the maps treated here, longitudes had to be estimated indirectly through travelled distances and directions. Real longitude determinations will appear in Chapter 4.

3.3 The moving pole in the sky

When describing his work with the star catalogue Brahe (1598) gives the following piece of information:

”For some particularly important stars, 100 altogether, we have … derived the right ascensions and declinations, and referred these to two secular years (namely 1600 and 1700), making it possible by a proportional calculation to derive values valid for epochs in between.”

This information, and later the catalogue itself, points out a fundamental problem appearing when using star coordinates for calculating coordinates on the Earth: All star coordinates undergo a systematic and gradual change with time, a phenomenon known since antiquity as *precession*.

The precession means that the Earth’s rotational axis, together with the Earth itself, continuously changes its direction (but not its inclination) in space, so that the whole equatorial coordinate system on the celestial sphere is moving. The process is such that the Earth with its rotational axis traces out a cone in space. The intersection of the rotational axis with the celestial sphere, i.e. the celestial pole, thus moves in a circle on the celestial sphere, the radius of the circle being equal to the inclination of the Earth's axis, a full revolution taking 26 000 years. The origin of the phenomenon was not known at the time of Brahe but later Isaac Newton (1687), the English mathematician, physicist and astronomer, showed it to be caused by the gravitational forces of the moon and the sun acting on the rotating Earth. The whole behaviour is identical to that of a spinning top; in that case it is the gravitation of the Earth that acts on the rotating body.

Now, the rate of the precession was found by Brahe to be 51″/year (modern value 50″/year corresponding to a full revolution in 26 000 years), i.e. nearly 1′ per year. After some years this becomes a quite considerable amount. Brahe therefore constructed his table of important stars in such a way that the user of the table should be able to easily correct the declinations (and right ascensions) for the precession.

Using Brahe's (1602) table, the effect of the precession upon latitude determination can be studied. Imagine someone measuring a latitude for Bure's map of 1626 the year before its publication, observing the pole star (Stella Polaris). From Brahe's table one can find that precession during the years elapsed since 1600 will affect the declination of the star by 9′. If this is not taken into account, the declination will be in error by that amount. Inserting such an erroneous declination into (3-3) will yield a latitude with the same error of 9′. This is twice as large as the standard deviation in the latitude determination as found in Section 3.2.

Imagine further someone measuring a latitude for the first Baltic Sea chart of 1694 the year before its publication, observing the same star. From Brahe's table and formula (3-3) one may conclude that ignoring the precession will cause an error in latitude of no less than 32′, more than half a degree. No other phenomenon has such a large effect on the determination of latitudes on the Earth.

By repeatedly observing the declinations of stars, the English astronomer James Bradley (1748) discovered a small periodical variation in the precession, known as nutation. Its origin is a variation of the moon's orbit, and its period is 18 ½ years; it turned out to be necessary to correct also for that when latitude determinations became increasingly accurate in the 1700s.

3.4 New observatories at Paris and Greenwich

In 1667, nearly 100 years after the foundation of Brahe's observatory at Uranienborg, an astronomical observatory was founded in *Paris*. Here the French astronomer and geodesist Jean Picard started working on a project of mapping France. He introduced the use of *telescopes* when making astronomical and geodetic measurements. This enabled him, among other things, to determine a quite accurate value of the Earth's radius. When the observatory had been established, one of the first tasks was to determine the positions of Paris and Uranienborg relative to each other. For this purpose Picard travelled to the island of Ven, in 1671. Arriving there he realized that the observatory had, sadly enough, been demolished into ruins by order of the Danish king after Tycho Brahe had left it, and the island itself had recently been ceded by Denmark to Sweden.

Picard, together with the Danish astronomer Ole Rømer, now determined the latitude of the ruins of Uranienborg. This was made in two different ways, published by Picard (1680). First, the latitude was determined using a circumpolar star, through (3-1), resulting in 55°54'40" (after correction for refraction). Second, the latitude was determined using a star close to zenith (where refraction is zero) with its declination from Paris, through (3-3), resulting in 55°54'15". Both values are slightly larger than those of Brahe (corrected for refraction), and the average comes close to the modern value; see Table 3-1. The unpublished value of Brahe in the table is cited by Picard as being the result of the later and thereby best part of Brahe's measurements. The second of Picard's values was later adopted as a starting value in the Swedish coastal triangulation around the Baltic Sea (Chapter 5).

A few years after the establishment of the observatory in Paris, a similar astronomical observatory was founded at *Greenwich* outside London, in 1675. This was more aimed at improving navigation at sea. Here the English astronomer John Flamsteed, after having determined the latitude of the observatory, spent most of his years with producing a new star catalogue. This catalogue of Flamsteed (1725) had an accuracy one order of magnitude better than Brahe's and thus superseded his. In Paris the French astronomer and geodesist Jacques Cassini (1740), based on the determination of the latitude of that observatory, also produced a catalogue of stars and other celestial objects. The new star catalogues from Greenwich and Paris were soon used for determining latitudes for a new map of Sweden and Finland (Chapter 4).

4. Moons and longitude: General maps of the third generation

4.1 The Jupiter moons at Uppsala

The basic principle for finding the *longitude* of a station with respect to a zero meridian at an observatory is quite simple. The longitude is equal to the *difference in local time* between the station and the observatory. When the principle is to be applied in reality things get more complicated. There are two problems to be handled. The first one is how to determine the local times; even if you have constructed good clocks they have to be set. The second one is how to find out the two local times at one and the same instant for two widely separated points on the Earth.

The easiest way to determine the local time is to use the *sun.* When the sun is in the meridian in the south, local solar time is by definition precisely 12 hours (ignoring here the difference between true and mean solar time). This can be used for setting a sun dial or a more advanced clock, at the observatory giving the time T_0, and at the station giving the time T.

A more accurate way of determining the local time is to use the meridian transit of a *star.* This means using sidereal time, defined by the Earth's rotation relative to the stars rather than to the sun. Such a method was not applied until later.

The longitude λ of the station relative to the observatory with the zero meridian now becomes

$$\lambda = T - T_0 \tag{4-1}$$

This is the difference in local time between the station and the observatory. Normally the time difference is then converted to angular units (1 hour = 15°).

Now, to measure the two local times in (4-1) at one and the same instant one would need a well-defined event that could be *observed simul-*

taneously from the two positions. Such events were the *eclipses of the moons of the planet Jupiter* (on rare occasions also eclipses of our own moon or the sun).

When the French astronomer and geodesist Jean Picard in 1671 redetermined the latitude of Uranienborg (Section 3.4), he also determined its longitude relative to Paris according to the above principles, using the eclipses of the moons of Jupiter. This can be considered as the first reasonably accurate determination of a longitude; it was made possible by the introduction of the telescope and the recently invented *pendulum clock*. We will comment on the result in Section 4.3. Picard's co-worker at Uranienborg, the Danish astronomer (and later mayor of København) Ole Rømer, then accompanied Picard to Paris, continuing to observe Jupiter's moons. There he found that certain irregularities in the observed times of the eclipses of the moons were dependent on the varying distance between the Earth and Jupiter. From this, Rømer (1676) could make the very first determination of the velocity of light.

In 1739 the Swedish geophysicist and geodesist Anders Celsius – today mostly known for his temperature scale – founded an astronomical observatory at the University of *Uppsala;* see Figures 4-1 and 4-2. This observatory was to play a special role in the early work with longitude determinations. Celsius (1741) writes:

"During the work with my astronomical observations I have especially paid attention to such phenomena, whereby the positions of places here in Sweden as well as abroad, with respect to their longitude east or west of the Uppsala meridian, may be more accurately marked on the maps than hitherto."

One of Celsius' research assistants here, Pehr Wargentin, an astronomer and statistician, now specialized in Jupiter's moons.

Jupiter has four large moons, discovered by Galilei when he for the first time used a telescope to study the sky. The four moons move in orbits around Jupiter, the innermost moon (Io) having the shortest period, only 42 hours. This means that an eclipse occurs at least every second day, thus providing frequent and well-defined events useful for longitude determination provided the planet can be clearly observed (which

Figure 4-1. The observatory of Uppsala at the time of Celsius, when it was established as a Nordic zero meridian. (Engraving by F Akrel 1769.)

it often cannot). However, each moon, in addition to being governed by the gravitation of Jupiter, is perturbed by the gravitations of the other moons. Hence the orbital motions of the moons become irregular, their periods not being constant. This makes it difficult to predict when the eclipses will occur, something that was essential for the determination of longitudes.

Wargentin systematically studied these irregularities in the motions of Jupiter's moons. Using an intuitive statistical method, Wargentin (1741) succeeded in computing a set of tables for the Jupiter moons, predicting the times of their future eclipses. (Sadly enough his tables were stolen on a journey just before their printing, so he had to spend nearly two years reconstructing them!) Such tables, although rather inaccurate,

Figure 4-2. The old Celsius observatory in Uppsala today.

had already been produced at the observatories of Paris and Greenwich. It soon turned out, however, that the tables produced by Wargentin at Uppsala were superior in accuracy to those of both Paris and Greenwich. From now on Wargentin's tables became a sort of international standard; repeatedly improved versions by Wargentin himself were included in the astronomical tables of Paris, Greenwich and Berlin for several decades.

Wargentin spent nearly his whole life observing Jupiter's moons and improving his tables. When Jupiter's moons had been observed for longitude purposes at some station in Europe, the largest probability of finding corresponding observations was at the observatory of Uppsala, later Stockholm when Wargentin moved there. Because of this, Uppsala, and later Stockholm, became an international key observatory for longitude determinations in their early days.

Looking back we may describe the whole procedure of *longitude determination, using observations of moons in combination with sun (star) observations,* in the following three steps:

1. Determine the time at an observatory using the meridian transit of the sun (or a star). Set a clock according to that.

2. Determine the time at an arbitrary station using the meridian transit of the sun (or a star). Set a clock according to that.

3. Observe, using the clocks, the times for an eclipse of a Jupiter moon (or for a lunar or solar eclipse) at both the observatory and the station. The time difference obtained is equal to the longitude of the station relative to the observatory, according to (4-1).

Item 3 above may be replaced by a simplified version requiring only one time observation but yielding a lower accuracy:

3*. Observe, using the clock, the sidereal time for an eclipse of a Jupiter moon at the station, and take the corresponding time at the observatory from a table of predictions valid there. The time difference, again, is the longitude of the station relative to the observatory.

4.2 The first maps based also on moon observations

Hitherto all maps had been based on determinations of latitudes, whereas longitudes had not been possible to measure. They had only been roughly estimated using indirect methods. The activities at the new observatory of Uppsala now opened up the possibility to construct maps based also on longitude determinations. Since longitudes were more complicated to measure than latitudes, however, the longitude determinations had to be limited to a small number of stations.

The first Nordic longitude measurements were carried out between København, Uppsala and Torneå. In København there was the observatory on top of the church tower of Rundetårn [Round tower], founded already by one of Brahe's assistants, the Danish astronomer Christian Longomontanus. Using lunar eclipses observed both there and at the observatory of Uppsala, Celsius (1741) determined the longitude difference

Figure 4-3. Map of Sweden, Finland and the Baltic Sea, including small parts of Denmark and Norway (Biurman, 1747), the first map based also on longitudes from moon observations.

between København and Uppsala. In Torneå in the far north a small private observatory had been erected by the Swedish-Finnish geodesist Anders Hellant, an assistant during the French arc measurement there (see Chapter 5). Using eclipses of Jupiter's moons observed both there and at the observatory of Uppsala, Celsius (1743) determined also the longitude difference between Torneå and Uppsala. These longitudes were included as an important part of the basis for a new *map of Sweden, Finland and the Baltic Sea,* the *first one based also on longitude determinations.* The map was constructed by the Swedish map engraver Georg Biurman (1747), who had earlier been travelling and working together with Celsius; it is shown in Figure 4-3.

The longitudes of Celsius used for Biurman's map, together with some comparisons, are given in Table 4-1. While the estimated longitudes on the earlier maps show errors of one or several degrees, the first measured longitudes applied for the new map show errors of only some minutes. Although very few, the measured longitudes were essential for the mapping. Because of their geographical distribution, widely separated in southwest-northeast, they contribute considerably to correcting and controlling the map.

In the following years another five longitude stations were established, all of them with the use of Jupiter's moons. The total of 8 longitude stations, mainly at observatories or universities, together with their longitudes, are listed in Table 4-2. From their errors, calculated as measured minus modern longitudes, we find a standard deviation in longitude of

Table 4-1. Longitudes (in degrees and minutes) according to the determinations of Celsius used for the map of Biurman (1747), and comparisons with the earlier maps of Ortelius (1570) and Bure (1626) as well as with modern values.

Station	1570 map	1626 map	1747 map (Celsius)	Modern
Torneå (Tornio)	9° 05′	5° 19′	6° 34′	6° 31′
Uppsala	0 00	0 00	0 00	0 00
København	- 5 30	- 6 35	- 4 58	- 5 04

Table 4-2. Longitudes (in degrees and minutes) according to determinations relative to Uppsala made in the 1740s and 1750s, and their errors. Stations are ordered from north to south.

Station	Data source	Measured	Modern	Error
Vadsø	Hellant (1752)	12° 14′	12° 07′	7′
Torneå (Tornio)	Celsius (1743)	6 34	6 31	3
Härnösand	Schenmark (1754)	0 15	0 18	- 3
Åbo (Turku)	Gadolin (1753)	4 31	4 38	- 7
Uppsala	---	0 00	0 00	-
Stockholm	Wargentin (1761)	0 25	0 25	0
København	Celsius (1741)	- 4 58	- 5 04	6
Greifswald	Mayer (1756)	- 4 03	- 4 14	11

$$\sigma_\lambda = 6'$$

This value corresponds to close to half a minute in time. It also corresponds to an uncertainty of about 5 km on the Earth's surface. This means that the longitudes now had reached an accuracy comparable to that of the latitudes one century ago (Section 3.2).

At the same time the accuracy of latitude determinations had increased, starting with Horrebow (1735) in København and Celsius (1739) at Uppsala. This was due to new instruments with telescopes, new star catalogues from Greenwich and Paris, and new methods of calculating disturbing effects. In Table 4-3 are listed the same stations as in Table 4-2, but now with their latitudes. The errors here yield a standard deviation in latitude of only

$$\sigma_\varphi = 0.1'$$

corresponding to 200 m on the Earth's surface. Thus a considerable gap in accuracy remained between latitude and longitude determinations, together known as *astronomical positioning*.

Table 4-3. Latitudes (in degrees and minutes) according to accurate determinations made in the 1730s, 1740s and 1750s, and their errors. Stations are ordered from north to south.

Station	Data source	Measured	Modern	Error
Vadsø	Hellant (1750)	70° 04.7′	70° 04.6′	0.1′
Torneå (Tornio)	Maupertuis (1738)	65 50.8	65 50.9	- 0.1
Härnösand	Schenmark (1754)	62 38.0	62 37.9	0.1
Åbo (Turku)	Gadolin (1753)	60 27.2	60 27.1	0.1
Uppsala	Celsius (1739)	59 51.7	59 51.7	0.0
Stockholm	Wargentin (1759)	59 20.5	59 20.5	0.0
København	Horrebow (1735)	55 41.0	55 40.9	0.1
Greifswald	Mayer (1756)	54 04.4	-	-

The Härnösand station in Tables 4-2 and 4-3 has a quite special background. In the 1600s the Paris observatory had determined a very approximate value of the distance to the sun. In the early 1750s French scientists now wanted to re-determine the distance to the sun with better accuracy. This could be accomplished by making simultaneous observations from an extremely southern station, decided to be Cape Town in South Africa, and a few extremely northern stations on the same longitude as Cape Town. The optimum northerly stations turned out to be Stockholm and Härnösand! The results, first published by Wargentin (1756), soon proved to be less accurate than expected, but renewed attempts with better methods some years later were quite successful. An important requirement here was the accurate knowledge of the coordinates of the observation points.

The most remarkable of the stations in Tables 4-2 and 4-3 is Vadsø, situated on the coast of the Arctic Sea. Its position was determined by Hellant (1750, 1752); again northern Norway was measured by the Swedes, this time just a few years before the border there was defined. Hellant had specialized in performing geodetic and geophysical measurements north of the Arctic circle. It was certainly not an easy task transporting scientific instruments in the cold winter darkness through the vast wilderness in the north. Hellant (1750) reports:

”Our sledges, or rather half-boats [Laplander’s sledges], by which you go with reindeer, swam through the snow when I went along the Teno river to Norway. … At the winter solstice at Vadsø you see stars with the naked eye all through the day.”

In addition to the accurate stations of Tables 4-2 and 4-3, latitudes could now be measured anywhere for general mapping purposes within some 0.5′. Longitudes, on the other hand, were so complicated to measure that they were restricted mainly to the few stations in the tables. Elsewhere longitudes still had to be indirectly determined from travelled distances and directions, but there were now at least a set of longitude stations putting constraints on these measurements. The improved positioning led to the first series of provincial maps showing latitudes and longitudes.

4.3 Connections to Paris and Greenwich

As long as one keeps to making a map over the Nordic area it is sufficient to relate the longitudes to a Nordic zero meridian like Uppsala. In a more global perspective, important for marine charts, the Nordic longitudes need to be related also to some more internationally recognized initial meridian, i.e. Paris or Greenwich.

In the beginning, longitudes were determined successively from Paris via Uranienborg to København and Uppsala; see Table 4-4. It started, as mentioned earlier, with Picard (1680) together with Rømer determining the longitude difference between Paris and Uranienborg. In connection with that, the difference between Uranienborg and København was found through various local measurements. Later Celsius (1741), as also mentioned earlier, determined the longitude difference between København and Uppsala.

In 1748 a new astronomical observatory was founded on a hill just outside *Stockholm* by the Royal Swedish Academy of Sciences; see Figures 4-4 and 4-5. Wargentin moved there from Uppsala, continuing and intensifying his internationally renowned studies of Jupiter’s moons. Soon

Table 4-4. Successive longitude differences (in degrees and minutes) from Paris according to determinations between observatories made up to the 1750s, and their errors.

Observatories	Data source	Meas.	Modern	Error
Paris – Uranienborg	Picard (1680)	10° 32.5′	10° 21.6′	10.9′
Uranienborg – København	Bartholin (1672)	- 7.2	- 7.2	-
København – Uppsala	Celsius (1741)	4 57.5	5 03.9	- 6.4
Uppsala – Stockholm	Wargentin (1761)	25.0	25.0	0.0
Paris – Stockholm	Sum of above	15 47.8	15 43.3	4.5
Paris – Stockholm	Wargentin (1761)	15 42.5	15 43.3	- 0.8

he determined also the longitude difference between Uppsala and Stockholm; see again Table 4-4.

After 12 years at the Stockholm observatory Wargentin had collected a large number of observations of the Jupiter moons. From the observatory in *Paris* he received their Jupiter observations during the same period. Wargentin (1761) writes:

”I have, during the last 12 years, had the opportunity to make a number of reliable observations at the observatory in Stockholm, with the purpose of finding out its geographical longitude. … Since the observations of the first or innermost moon of Jupiter give the most reliable values of the longitude, without troublesome calculations, I will keep to them in order to obtain the difference between the meridians of Paris and Stockholm.”

Wargentin’s result for the longitude of Stockholm relative to Paris is λ = 15°42′30″. This is 5′ less than the earlier longitude determined piece by piece. From Table 4-4 we note that this discrepancy is almost wholly due to errors in those older determinations. In the same table we find that the error in Wargentin’s longitude determination amounts to less

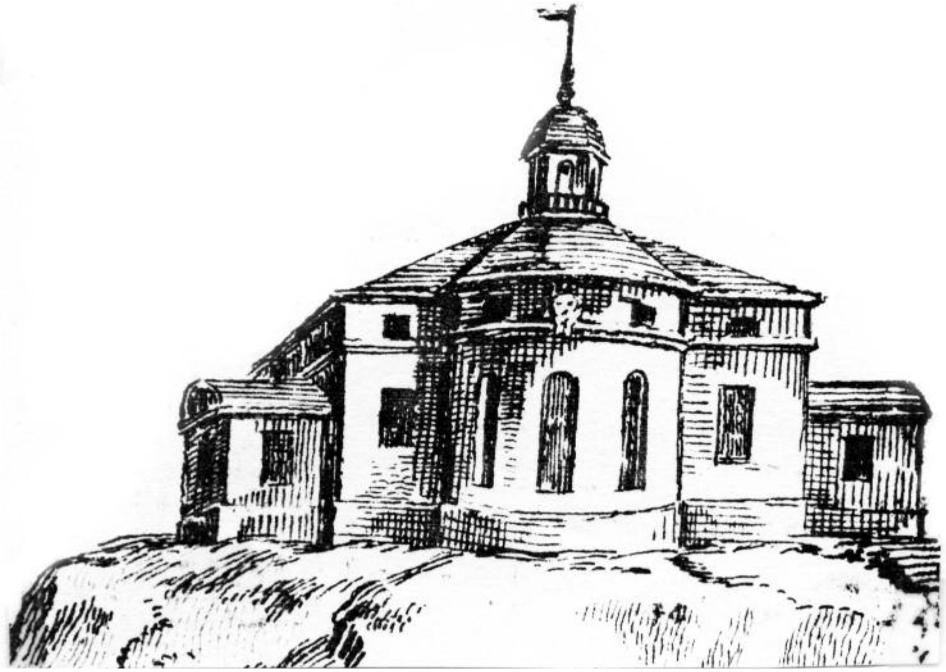

Figure 4-4. The observatory of Stockholm, specialized in longitude determination (Wargentin, 1761).

than 1′. It is interesting to compare this with the error according to Table 3-1 in the latitude determination by Brahe at Uranienborg one and a half century earlier. The errors are of the same order. This clearly illustrates the difficulties in finding the longitude, where times are involved, compared to latitude, where angles are involved.

After another 12 years Wargentin (1773) had improved the longitude difference between Paris and Stockholm, using the Jupiter moons, to λ = 15°43′15″. This yields an error as small as 0.1′, probably the world's best longitude determination at that time; see Table 4-5. At the same time the Swedish-Finnish-Russian astronomer Anders Johan Lexell (1773) had calculated the longitude difference from a solar eclipse, giving an error of 0.5′.

Wargentin (1777) now turned his interest towards *Greenwich*; see Table 4-5. Using 10 years of observations of Jupiter's moons from the ob-

Figure 4-5. The old observatory of Stockholm today.

servatories of Stockholm and Greenwich he found a longitude difference between them of λ = 18°05′15″. This yields an error of 1.7′, clearly less accurate than the difference between Stockholm and Paris. Hence this error must primarily depend on the observations at Greenwich.

Combining the Stockholm longitude relative to Greenwich with that relative to Paris, Wargentin (1777) could determine the important longitude difference between the observatories of Greenwich and Paris; see again Table 4-5. Actually, the head of the Greenwich observatory, the British astronomer Nevil Maskelyne, known for the creation of the Nautical Almanac, had asked Wargentin to calculate the longitude difference between Greenwich and Paris. Maskelyne (1787) writes:

"In the year 1776, I requested the late Mr. Wargentin, the learned secretary of the Royal Academy of Sciences at Stockholm, and author of the improved tables for computing the eclipses of Jupiter's satellites, who

Table 4-5. Longitude differences (in degrees and minutes) between Paris, Greenwich, Stockholm and København according to determinations made in the 1760s and 1770s, and their errors.

Observatories	Data source	Meas.	Modern	Error
Paris – Stockholm	Wargentin (1773)	15° 43.2′	15° 43.3′	- 0.1′
Greenwich – Stockholm	Wargentin (1777)	18 05.2	18 03.5	1.7
Greenwich – Paris (partly via Stockholm)	Wargentin (1777) Maskelyne (1787)	2 21.2	2 20.2	1.0
Paris – København (partly via Stockholm)	Bugge (1779)	10 16.6	10 14.4	2.2

collected observations of them from the principal observatories of Europe, … to inform me what difference of meridians of Greenwich and Paris resulted from my last ten years observations of the eclipses of the first satellite of Jupiter compared with those by M. Messier at Paris. In the answer which he favoured me with ... he deduced the difference of meridians of the Royal observatories of Greenwich and Paris ... from a comparison of mine and the Parisian observations, with the intermediate help of his own made at Stockholm 9 m 26 s [2° 21.5′]; and from the whole he inferred the difference of meridians to be 9 m 25 s [2° 21.2′]."

Wargentin's final value for the difference shows an error of 1.0′. It is quite interesting that one of the official values at this time of the longitude difference between the fundamental observatories of Greenwich and Paris was determined in Stockholm, through Wargentin's observations of his beloved Jupiter moons there.

Also when the Danish astronomer and geodesist Thomas Bugge (1779) determined the longitude of the København observatory relative to Paris he partly did so via Stockholm; see Table 4-5. In addition to the Jupiter moons Bugge also used a solar eclipse, giving practically the same result.

A few years later there was a break-through for the *chronometer,* a *portable clock* that could bring Greenwich time across the sea, invented by the English carpenter John Harrison (and for a long time combated by Maskelyne). The chronometer tolerated smooth movements; this suddenly made possible the determination of longitudes on board ships during navigation and in harbours. A zero meridian time could now simply be *transported* anywhere and compared with the local time from sun (or star) observations. However, while the chronometer was constructed to be transported on sea, it did not work very well when transported on land.

Referring to the longitude formula (4-1) we may here give an overview of the two principle methods to acquire *coincidence in timing at two different places for longitude determination:*

A. Observe a well-defined event in the sky (an eclipse of a Jupiter moon) from both places, and afterwards send a letter with the time from one place to the other (this chapter).

B. Transport the time with a portable clock (chronometer) from one place to the other (next chapter). Later on time could also be transported immediately by telegraph.

On the whole, longitudes continued to be difficult to determine, except on board ships on the sea. On land, on the other hand, a new method of accurate positioning entered the scene: triangulation. This, combined with the astronomical positioning at observatories, would become the foundation of future mapping, as will be seen in the following two chapters.

5. Stars, clocks and triangles along coasts: Marine charts

5.1 Solving an international controversy: The arc at the Arctic circle

In the 1730s an international scientific controversy with wide implications arose concerning the shape of the Earth. In England Newton had, based on his theories of gravitational and centrifugal forces, arrived at the conclusion that the Earth must be a body somewhat flattened at the poles. In France Cassini at the Paris observatory had, based on his geodetic measurements across the country, arrived at the conclusion that the Earth must be a body somewhat flattened at the equator, thus contradicting Newton's theories. In order to solve the problem the French Academy of Sciences decided to organize two scientific expeditions, one to the south, close to the equator, and one to the north, as far north as possible.

By the time the plans for the northern expedition were being discussed in France, Celsius arrived there from Sweden. He was making a study tour to European universities and observatories, and he happened to arrive in Paris at the right moment. Following a proposal from Celsius, the French Academy of Sciences decided to send the northern expedition to northern Sweden (now Sweden and Finland), more specifically to the area of Torneå at the end of the Gulf of Bothnia, close to the Arctic circle. Celsius had never been so far north himself, but his grandfather, the astronomer and geographer Anders Spole, had been there together with the mathematician Johan Bilberg to make the first scientific studies of the midnight sun; Bilberg (1695) had published a report on this. Celsius now became a member of the expedition, which was headed by the French physicist Pierre Louis Moreau de Maupertuis and his mathematical colleague Alexis Claude Clairaut.

The main task for Maupertuis' expedition was to perform a *meridian arc measurement*, i.e. to determine the distance as well as the latitude difference between the end points of a meridian arc. A comparison of such a result in the north with a corresponding result from an arc in the south,

Figure 5-1. Torneå (Tornio) church close to the Arctic circle, southern end point in 1736 of the French arc measurement, led by Maupertuis.

in France or at the equator, would give information on the flattening of the Earth. For an Earth flattened at the poles a meridian arc of a certain latitude difference will be longer closer to the pole, because of the smaller curvature there, and shorter closer to the equator, because of the larger curvature there. For an Earth flattened at the equator the relation will be the opposite.

The latitude difference of the meridian arc could be found by determining the latitudes of the end points through star observations. The southern end point was Torneå church (Figure 5-1), on the coast of the Gulf of Bothnia, and the northern end point was the mountain Kittisvaara, almost 1° to the north.

The distance between the end points, on the other hand, was far too long to be measured directly; it had to be found using a special method, *triangulation.* The idea of triangulation stems from Mercator's teacher Gemma Frisius 1533; it was tried in a primitive form by, among others, Tycho Brahe. However, it was through Picard's works in France since 1669 that the method of triangulation could be developed into something useful.

Triangulation now was applied in the following way. First, a comparatively short distance, a baseline, was measured with rods on the ice of the Torne river. Maupertuis et al (1738) vividly describe the problems thereby:

"Judge what it must be to walk in snow two foot deep, with heavy poles in our hands, which we must be continually laying upon the snow and lifting again; in a cold so extreme, that whenever we would take a little brandy, the only thing that could be kept liquid, our tongues and lips froze to the cup, and came away bloody."

Next, horizontal angles were measured in a network of triangles, the sides of the triangles being sight lines between stations on hills and mountains along the Torne river, all the way from the southern end point to the northern one. Included in this network were the end points of the baseline. Finally, using trigonometry, the distance between the southern and the northern end points of the meridian arc could be computed from the length of the baseline and the angles in the triangulation network.

Knowing now the distance as well as the latitude difference of the meridian arc, its curvature could be computed. Comparing this result of the northern expedition with a corresponding result in France, and later on with the result from the equatorial expedition, Maupertuis et al (1738) found that the meridional curvature of the Earth is smaller closer to the pole and larger closer to the equator. From this they concluded that the Earth is flattened at the poles; in the words of Maupertuis et al (1738):

"The length of the arc of the meridian intercepted between the two parallels that pass through the observatories of Torneå and Kittis is 55 023 ½ toises [107 243 m]. The amplitude of this arc being 57'27", the degree of the meridian at the Polar circle is greater by 1 000 toises [1 949 m] than it should be according to Mr. Cassini. … Whence it is evident that the Earth is considerably flattened towards the poles.”

This conclusion was confirmed by the gravity measurements made by the expedition, analyzed by Clairaut (1743). Although the accuracy of Maupertuis’ result later turned out to be clearly less than he himself had imagined, the total result of the expedition supported the theories of Newton.

5.2 The triangles across the Baltic Sea

The result of Maupertuis’ expedition to the north not only had scientific but also practical consequences. In France triangulation had been applied by Cassini in order to construct a foundation for an accurate map of the whole country. The results from the north now brought about a partial remeasurement of the French triangulation, using one of the expedition’s two angle instruments, to improve the foundation of the map. The triangulation network of France was completed and published by Cassini’s son and successor at the Paris observatory, César Francois Cassini de Thury (1744). This was the first triangulation in the world for constructing a national land map. The map itself, in a large number of sheets, took another half a century to produce.

The other of the two angle instruments of Maupertuis’ expedition was used in another triangulation for accurate mapping: a *triangulation across the Baltic Sea between Sweden and Finland, via the Åland Islands.* In the Nordic area this was the *first triangulation for official mapping.* Moreover, it appears to have been the first triangulation in the world for marine purposes, resulting in nautical charts as well as a land map.

The Åland Islands include an extensive archipelago comprising thousands of islands and skerries. Between and across these islands ran the important ”Post route”, used for transporting not only mail and goods but also diplomats and other people travelling between the western and eastern parts of northern Europe. This area certainly was in need of a more reliable mapping. The triangulation across the Baltic Sea was organized as a cooperation between the Royal Survey Office, the University of Åbo and, in the background, the University of Uppsala. The triangulation was carried out between 1748 and 1752 under the lea-

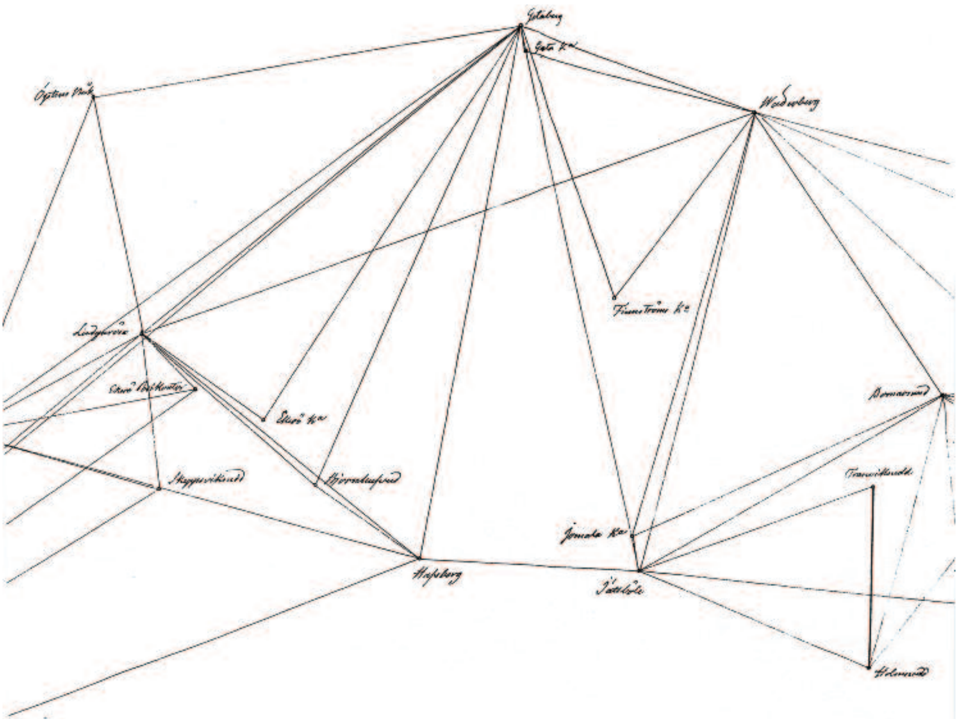

Figure 5-2. The central part of the triangulation network across the Åland Islands, established 1748 – 1752 (Gadolin, 1757; Hällström, 1815). Some of the stations are also modern triangulation stations.

dership of the Swedish-Finnish geodesist Jacob Gadolin, a former student of Celsius. The triangulation network covered 3 ½ ° in longitude, approximately along the latitude 60°, from Väddö in eastern Sweden across the Åland Islands to Åbo in south-western Finland; see Figure 5-2. The principle for determining coordinates through triangulation may be described as follows, with this triangulation as a typical example.

The first step in the triangulation is to make an *astronomical determination of latitude and longitude* of one station. This is necessary in order to position the whole triangulation network on the Earth. The astronomical station selected in our case was Åbo cathedral.

The second step is to make an *astronomical determination of an azimuth*, the azimuth being the horizontal angle between the direction towards the north and one of the sides in the triangulation network. This is necessary in order to orientate the whole triangulation network on the Earth. In this case the azimuth of a triangle side along the west coast of Åland was determined.

The third step, and the most time-consuming work, is to measure all *horizontal angles* between the triangulation stations in the network. To make such measurements one first has to select suitable stations with good sights towards other stations, and on each station erect some kind of signal to be measured against from the neighbour stations. In our case of the Åland Islands the triangulation stations could be selected in a very special way, namely among the beacon cliffs there. They constituted an ancient warning system in case of threatening attacks. Not only did these cliffs have excellent sights between them, but they were also already monumented with signals in the form of wooden beacons to be set fire on in case of emergency. In addition some churches with towers as well as skerries with nautical constructions could be used for triangulation. In total the triangulation network comprised some 40 stations, with sight lines of up to 40 km (in one case 70 km).

The fourth step is to measure the *length of a baseline*. When selecting a suitable baseline one should look for a reasonably flat surface close to sea level. In the case of the Åland Islands there appeared an ideal and almost unique possibility: The baseline could be measured directly on the ice of the sea. A 10 km distance was measured on a part of the ice-covered sea at Åland surrounded by sheltering islands, thereby reducing possible movements of the sea ice.

The final step is to compute the coordinates of all triangulation stations. Starting from the latitude and longitude of the astronomical station, and applying trigonometry to the angles and the baseline in the net of triangles, the latitudes and longitudes of all triangulation stations can be found. The calculations are complicated by the curvature of the Earth, but we leave that aside until Chapter 6.

Let us summarize the whole *procedure of triangulation* in the following five steps:

1. Determine astronomically the latitude and the longitude of one station, using the principles described in Chapters 3 and 4.

2. Determine astronomically the azimuth of (at least) one triangle side.

3. Measure all horizontal angles in the triangles of the network.

4. Measure the length of (at least) one baseline, also being a side of a triangle in the network.

5. Starting from the astronomical station in item 1 and the azimuth in item 2, and using the angles in item 3 and the length in item 4, calculate the latitude and the longitude of each station in the network.

The resultant coordinates of the triangulation across the Baltic Sea and the Åland Islands were presented by Gadolin (1757). These coordinates, together with other ones from connected triangulations performed later, served as a foundation for the construction of the first accurate *nautical chart,* produced by the naval officer and hydrographer Johan Nordenankar (1783); see further next section. The coordinates were also used for constructing a *land map,* produced by the land surveyor Eric af Wetterstedt (1789).

Let us now investigate the results of this historical triangulation. When doing so we utilize the recomputation of the whole triangulation executed by the Finnish-Swedish geodesist and cartographer Carl Peter Hällström (1815), using improved mathematical methods. For the investigation 13 main stations are selected, reasonably distributed within the triangulation network and having long sight lines to several neighbour stations. These stations are listed in Table 5-1. The first three of them are situated in Finland, the following nine ones on Åland, and the last one in Sweden. For each station are given its measured and modern latitude, in the upper part of the table, and its measured and modern longitude, in the lower part of the table. These coordinates need some explanation before we enter into their analysis.

We already know the astronomical error in the starting point at the Åbo cathedral; it is given in Chapter 4. As we are not interested in that kind of error for investigating the accuracy of the triangulation we could

Table 5-1. Coordinates of the triangulation across the Baltic Sea via the Åland Islands in 1748 – 1752 (in degrees, minutes and seconds), and their errors. Stations are ordered from east to west. Data sources: Gadolin (1757) and Hällström (1815).

Station	Meas. lat.	Modern lat.	Error
Åbo (Turku) cathedral	60° 27′ 06″	60° 27′ 09″	- 3″
Prostvik beacon cliff	60 12 43	60 12 43	0
Korpo church	60 09 47	60 09 47	0
Kumlinge beacon cliff	60 14 54	60 14 52	+ 2
Ulversböte beacon cliff	60 06 30	60 06 28	+ 2
Bomarsund beacon cliff	60 13 05	60 13 04	+ 1
Väderberg beacon cliff	60 20 47	60 20 48	- 1
Jomala church	60 09 21	60 09 18	+ 3
Getaberg beacon cliff	60 23 09	60 23 10	- 1
Havisberg beacon cliff	60 08 45	60 08 45	0
Högsten nautical beacon	60 21 15	60 21 13	+ 2
Signilskär nautical cairn	60 12 09	60 12 04	+ 5
Stacksten [Väddö] beacon cliff	59 57 57	59 57 57	(0)

Station	Meas. long.	Modern long.	Error
Åbo (Turku) cathedral	22° 16′ 33″	22° 16′ 40″	- 7″
Prostvik beacon cliff	22 02 57	22 03 02	- 5
Korpo church	21 33 44	21 33 50	- 6
Kumlinge beacon cliff	20 47 24	20 47 05	+ 19
Ulversböte beacon cliff	20 33 52	20 33 46	+ 6
Bomarsund beacon cliff	20 13 50	20 13 40	+ 10
Väderberg beacon cliff	20 03 31	20 03 27	+ 4
Jomala church	19 56 59	19 56 59	0
Getaberg beacon cliff	19 50 33	19 50 40	- 7
Havisberg beacon cliff	19 45 22	19 45 19	+ 3
Högsten nautical beacon	19 27 30	19 27 13	+ 17
Signilskär nautical cairn	19 20 32	19 20 15	+ 17
Stacksten [Väddö] beacon cliff	18 50 18	18 50 18	(0)

eliminate it by assigning this station its modern coordinates, and then apply the same correction to all the other stations. In this way we also eliminate a principal difference between astronomical and modern coordinates to be discussed in Chapter 7. However, since the main part of the network has a rather weak connection to Åbo cathedral, it seems more relevant to assign the modern coordinates to the other end station, Väddö beacon cliff; this is a modern triangulation and satellite positioning station as well. Accordingly all the original measured coordinates of the triangulation stations have been shifted by a constant in latitude and another constant in longitude (the shift in longitude also contains a change of the zero meridian). These translated coordinates are the ones given in Table 5-1 as measured coordinates.

The modern coordinates in Table 5-1 are given in a system based on satellite positioning (see further Chapter 7). All stations have been identified on present-day detailed maps and charts, and their coordinates taken from there; some of the stations are also modern triangulation stations. For the initial station, the western end station Väddö beacon cliff, the coordinates have been checked against the satellite positioning performed there. We should also mention that the effect of different reference ellipsoids (see Chapter 6) used for the measured and modern coordinates, respectively, is negligible in this case.

The error of a measured coordinate in Table 5-1 is calculated as the difference between measured and modern coordinates. Starting with the latitudes we find that the standard deviation in latitude amounts to

$$\sigma_\varphi = 2'' \approx 60 \text{ m}$$

No systematic error can be found. Turning to the longitudes we notice some effects that do not appear quite random; they might be due to the design of the network with long sights and small angles in the longitudinal direction. Eliminating the small non-zero average of the errors (dependent on the choice of the initial station) we find that the standard deviation in longitude amounts to

$$\sigma_\lambda = 9'' \approx 140 \text{ m}$$

(bearing in mind that the length of 1″ of longitude at the latitude 60° is one half of the length of 1″ of latitude). Thus the overall uncertainty of the coordinates of the triangulation may be said to be about 100 m.

To be able to easily compare the above values with the earlier uncertainties of astronomical latitude and longitude determination in Chapter 4, we express the above standard deviations in minutes with decimals,

$$\sigma_\varphi = 0.03'$$

and

$$\sigma_\lambda = 0.15'$$

Comparisons with the corresponding values in Section 4.2 (there in the opposite order) reveal a tremendous increase in accuracy in longitude. The longitude accuracy has suddenly increased by a factor of 40! Moreover, the number of stations with known longitudes has increased considerably and could be further increased by expanding the triangulation. Also latitudes could now be found with high accuracy at a large number of stations in the same way. Triangulation thus meant a revolution in the determination of coordinates for mapping the Earth.

We may note here that Gadolin himself tried to check his triangulation by making astronomical latitude determinations on seven of the triangulation stations. The discrepancies turn out to have a standard deviation of 8″ (although with a somewhat skew distribution). As the standard deviation of the triangulated latitudes is only 2″ according to above, the discrepancies mainly reflect the uncertainty in the astronomical latitudes. This uncertainty is in good agreement with that of Section 4.2.

It is also interesting to compare the accuracy of this triangulation with that of the French arc measurement at the Arctic circle. In that case the total distance along the triangulation network was found to be slightly more than 100 000 m. The error in this was shown through a more accurate measurement by Svanberg (1805), treated in Chapter 6, to be about 50 m. This makes a relative error of 1 : 2000. In the Baltic Sea triangulation via the Åland Islands the total difference in longitude along the triangulation is 3 ½ ° or nearly 200 000 m. From Table 5-1 we find that a scale error in this quantity hardly can exceed 5″ – 10″ or some 100 m. This makes a relative error hardly exceeding 1 : 2000. Thus the triangulation of Åland seems to have been of the same quality as the one by the French expedition in the north.

A few years before the Åland triangulation a highly original but never-implemented method for positioning along coasts was put forward by the Swedish applied mathematician Jonas Meldercreutz (1741). He suggested using war-ships in very much the same way as satellites are used for positioning today; for a discussion of this see Chapter 8 (Section 8.3).

5.3 Triangulation along the coasts and the first nautical charts

Inspired by the successful triangulation and mapping of France, other countries decided to do the same. The first countries after France to decide on national triangulations for mapping were Sweden, including Finland, and Denmark, partly with Norway. The reason for the Nordic countries being so early was most probably the French arc measurement here, with the participation of Celsius, and the subsequent successful triangulation of Åland. The *Swedish-Finnish triangulation* was a coastal one designed for producing *nautical charts*. The *Danish and Norwegian triangulations* also covered the inland and were primarily aimed at producing *land maps.*

In Sweden the triangulation was performed following a decision of the parliament in 1756, right after the triangulation of Åland had been completed. Both Sweden and Finland have complicated coasts with extensive archipelagos, and there was an urgent need for charts useful for navigation. The triangulation covered all Swedish and Finnish coasts along the Skagerrak, the Kattegat, the Baltic proper, the Gulf of Bothnia, and the Gulf of Finland. The Finnish part of the triangulation was connected to the Swedish one both around the Gulf of Bothnia and across the Åland Islands. In addition, a part of the German coast along the Baltic, under Swedish sovereignty, was included. The triangulation was carried out, with interruptions, during a period of 30 years, 1758 – 1786. The project as a whole was put under the supervision of the Royal Academy of Sciences, especially its astronomer and statistician Pehr Wargentin at the Stockholm observatory, together with the Admiralty, which had sent for the mathematician Mårten Strömer from the Uppsala observatory.

The triangulation started along the west coast of Sweden, the first results being presented by the geodesist Nils Schenmark (1765). Uranienborg, remarkably enough, was selected as initial astronomical station, although ruined since almost two centuries. Its latitude was fixed at the value of Picard (1680), 55°54′15″, and its longitude provisionally at 0°. The resultant coordinates along the whole west coast were published by Schenmark (1774 & 1780).

Later on, after extending the triangulation to the east coast of Sweden and the coast of Finland, the observatory of Stockholm (Figures 4-4 and 4-5) became the main astronomical station of the triangulation. Its latitude was fixed at the value determined by Wargentin (1759) as the average of 59 star observations, 59°20′31″. From Table 5-2 we find that the error in his value is as small as 2″. The longitude of the observatory was put to 0°. Thus the Stockholm observatory became the zero meridian for Sweden and Finland; it also had a well-defined relation to the Paris meridian as described in Chapter 4. Azimuths and baselines were measured in different parts of the network. The special German part of the triangulation had to be measured separately; it relied on the astronomical coordinates of Greifswald.

Table 5-2. Latitudes (in degrees, minutes and seconds) of the national observatories as determined in the late 1700s and the early 1800s as a basis for the national triangulations, and their errors.

Observatory	Data source	Measured	Modern	Error
København	Bugge (1779)	55° 40′ 57.0″	55° 40′ 53.3″	3.7″
	Schumacher (1827)	55 40 52.6	55 40 53.3	- 0.7
Stockholm	Wargentin (1759)	59 20 31.3	59 20 33.0	- 1.7
	Cronstrand (1811)	59 20 34.8	59 20 33.0	1.8
	Selander (1835)	59 20 33.8	59 20 33.0	0.8
Oslo	Hansteen (1849)	59 54 43.7	-	-
Helsinki	Argelander (1837)	60 09 42.6	-	-

Figure 5-3. Nautical chart of the Åland Sea and its surroundings (Nordenankar, 1783), the first chart based on triangulation.

The resultant coordinates of the triangulation, except for the already mentioned ones along the Swedish west coast, were never published. What we have today is the end product, a set of considerably improved maps in the form of *nautical charts covering the entire Baltic Sea as well as the Kattegat and the Skagerrak;* these are the *first ones based on triangulation*. A few of the charts also could benefit from the Danish triangulation treated below. The charts, all of them in the Mercator projection, include depths where measured, but these data are in general very sparse. The first (dated) chart was the earlier mentioned one of the Åland Sea, extending southwards to Stockholm, issued by Nordenankar (1783); see Figure 5-3. The whole set of charts, known as Nordenankar's sea atlas, was issued during the years 1783 – 1791. It should be mentioned here that the longitudes on the charts are counted from *Ferro* (Hierro), one of the Canary Islands. In reality this means that the longitudes, via Stockholm, are determined relative to *Paris,* but that an arbitrary constant of close to 20° has been added. This was a common trick at that time in order to create positive longitudes all over Europe.

The accuracy of the Swedish-Finnish coastal triangulation is difficult to investigate, as much information and data are lacking. There are clear signs, however, that the accuracy cannot compete with the high quality of the Åland triangulation. It appears from Schenmark (1774) that astronomical positionings have been used as constraints in the triangulation network, indicating a need to strengthen the triangulation data themselves over long distances (the French instrument was no longer in use). The uncertainty of an astronomical latitude determination was about 8″ according to Section 5.2; the uncertainty in the triangulation should thus be considerably larger. This is confirmed by Hällström (1815), who claims that the Åland triangulation was superior to every part of the large Swedish-Finnish coastal triangulation. We will return to this matter in connection with the accuracy of the Danish triangulation below.

In Denmark the triangulation was performed following a decision of the Royal Academy of Sciences in 1761, soon after the Swedish triangulation had commenced. Denmark is a country favourable for measurements of this kind. The triangulation therefore covered the whole country. It was carried out during almost the same period as the Swedish-Finnish one, 1762 – 1792. The project was under the leadership of the

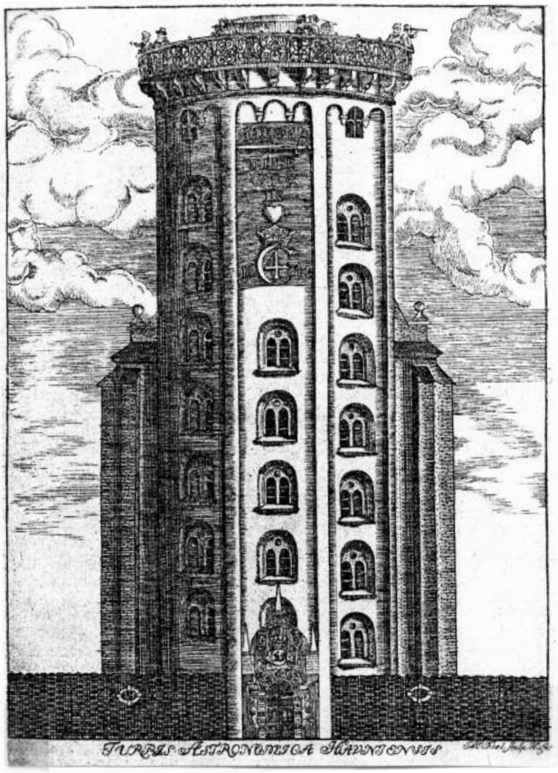

Figure 5-4. The observatory of København (Horrebow, 1735), fundamental astronomical station for several triangulations through the works of Bugge and Schumacher.

Royal Academy of Sciences, most of the time through the astronomer and geodesist Thomas Bugge at the København observatory.

The triangulation started around København. The København observatory (Rundetårn) served as the main astronomical station; see Figures 5-4 and 5-5. Its latitude was fixed at the value determined by Bugge (1779) as the average of 56 star observations, 55°40′57″; see also Bugge (1784). From Table 5-2 we find that the error in this value is 4″. The longitude of the observatory was put to 0°. Thus the København observatory became the zero meridian of Denmark; it also had a known relation to the Paris meridian as described in Chapter 4. An azimuth was determined in København, and baselines were measured in different parts of

Figure 5-5. The old observatory of København today.

the network. The resultant coordinates for the whole island of Sjælland, where København is situated, were published by Bugge (1779).

Except for Sjælland the resultant coordinates of the triangulation have not been published. What we have today is the end product, a set of considerably improved land maps covering the entire Denmark. The first map was one showing a part of Sjælland with København, issued by the Norwegian-Danish geodesist and cartographer Caspar Wessel (1768); this is the earliest map based on the Danish triangulation. The whole set of maps was issued during the years 1768–1805. It should be noted here that the triangulation also turned out to be useful as a basis for nautical charts, namely for the Danish part of Nordenankar's charts of the Baltic Sea including the Kattegat.

Wessel was for many years heavily involved in the triangulation calculations behind the mapping. These calculations inspired Wessel (1797) to a fundamental mathematical discovery: the geometric interpretation of complex numbers as quantities in a coordinate system. (The same discovery was later made also by Gauss, without knowing about Wessel's work.)

The accuracy of the Danish triangulation can be judged from an investigation performed by Bugge (1779) combined with conclusions from the Åland triangulation. In a similar manner as Gadolin on Åland, he checked the accuracy of the triangulation by making astronomical latitude determinations on eight of the triangulation stations. The discrepancies show a standard deviation of 20″ – 30″ (600 – 900 m), depending on what stations are considered acceptable. Since the standard deviation of the astronomical latitudes should be some 8″ according to Section 5.2 (or maybe slightly larger here), the discrepancies in this case should predominantly reflect the uncertainty in the triangulation. Thus the overall uncertainty of the coordinates of the Danish triangulation may be estimated at some 500 m. The discussion of the Swedish-Finnish triangulation above roughly indicates an uncertainty there of the same order as the Danish one (this is supported by the fact that the same kind of instrument was used in both triangulations).

In Norway triangulation was performed through a special organization, the Geographical Survey of Norway, founded in 1773 for that purpose. The efforts were concentrated on the southern part of the country, where Oslo is situated. The triangulation of this part was carried out during the years 1779 – 1813. The project was ultimately supervised from Denmark, by Thomas Bugge at the København observatory, but later on the whole leadership was taken over by the Norwegian geodesist Benoni Aubert.

The triangulation started inland, in the south-east, close to the Norwegian-Swedish border. The main astronomical station selected, and thereby also the zero meridian, was quite special in the absence of an observatory: the flag pole at the fortress of Kongsvinger. Azimuths and baselines were measured in different parts of the network, the baselines mostly on frozen lakes and fiords. The triangulation continued along the coasts, in the south-west and the south, until a complete loop around southern Norway had been closed.

No series of maps or charts resulting from the triangulation was issued at this time, but a single map covering the whole of southern Norway was issued by the Danish cartographer C I Pontoppidan (1785). This seems to be the first map partially founded on the Norwegian triangulation.

The accuracy of the Norwegian triangulation is difficult to estimate because of the lack of published numerical results. However, as the triangulation to start with was performed according to detailed instructions from Bugge, it seems reasonable to assume that its accuracy should have been comparable to the Danish one. This is partly supported by some internal comparisons made within the triangulation net at that time. Astronomical checks of the triangulation net also seem to have been made, but do not allow any general conclusions.

In summary: The estimated uncertainty of the Nordic triangulations during the second half of the 1700s is some 500 m, with exception for the early triangulation of Åland where the uncertainty amounts to only 100 m. Thus the Åland triangulation appears to have an accuracy nearly one order of magnitude better than the others. This is quite remarkable, but might be explained by a higher ambition as a pioneering work, together with a better instrument. Already Hällström (1815) writes:

”In the conviction that this measurement, as regards accuracy, is superior to all those that hitherto have been made in Sweden and Finland, I have considered it to deserve a new calculation. … It will always display the observer’s eminent care and effort to achieve the highest possible degree of reliability.”

The quality of the Åland triangulation may also be the reason for a peculiar interest in it by the Russian navy nearly 100 years after the triangulation had been performed; see further Chapter 8 (Section 8.2).

As shown in Section 5.2 triangulation meant a revolution in the determination of coordinates for mapping, especially in finding the longitude. The first map-makers using triangulated coordinates, Wessel (1768) in Denmark as well as Nordenankar (1783) and Wetterstedt (1789) in Sweden-Finland, all point out on their maps that these are based on ”trigonometric and astronomical observations”.

Finally we should mention that the Swedish ironworks proprietor Samuel Gustaf Hermelin, in cooperation with Hällström, issued a unified map of the Nordic countries, as well as a series of provincial maps of Sweden and Finland, where the position and shape of the coasts rested on the triangulations. Most of the inland parts, on the other hand, had to rely on other kinds of data.

5.4 Shipping clocks across the North and Baltic Seas

When making the charts of the Baltic Sea, triangulation could not be extended to Gotland and several other distant islands in the Baltic proper. To compensate for the lack of triangulation, the Swedish-Finnish geodesist and hydrographer Nathanel Gerhard Schultén (1801), by order of the King, made astronomical positionings for a number of islands there. Latitudes were determined in the traditional way, in this case observing the sun (with a sextant). Longitudes, however, were now determined by the use of a chronometer, the transportable ship clock invented by Harrison a few decades earlier. This was the very first *longitude chronometer expedition* in the Baltic Sea; it was made in 1800 using a hydrographic sailing-ship. The chronometer was set and controlled with the sun in Stockholm, while the local time at each island was found from the sun observations there. The voyage was normal except for a heavy storm south of Åland.

To briefly investigate the results of this expedition we have selected one station on each island visited by Schultén; they are listed in Table 5-3 together with the latitudes and longitudes obtained by him. The original longitudes have, thereby, been shifted from Ferro to Greenwich (based on his starting longitude at Landsort south of Stockholm).

Beginning with the latitudes we find from their errors, calculated as measured minus modern latitudes, a standard deviation of 0.6′. This shows that the uncertainty in astronomical latitude determination under simple field conditions was one order of magnitude larger than under observatory conditions (see Table 4-3 in Chapter 4). Turning to the longi-

Table 5-3. Coordinates of the ship expedition to islands in the Baltic Sea in 1800 (in degrees and minutes), and their errors. Stations are ordered basically from west to east. For explanation of asterisks see text. Data source: Schultén (1801).

Station	Meas. lat.	Modern lat.	Error
Öland: Böda	57° 15.2′	57° 14.4′	+ 0.8′
Gotland: Visby	57 38.5	57 38.3	+ 0.2
Fårö: Avanäs	57 56.8	57 57.6	- 0.8
Gotska Sandön	58 20.6	58 20.8	- 0.2
Östra Bogskär [S of Åland]	59 30.0	59 31.0	- 1.0
Utö [SE of Åland]	59 46.4	59 46.9	- 0.5
Dagö: Dagerort (Hiiumaa: Köpu)	58 54.6	58 55.0	- 0.4
Ösel: Svarverort (Saaremaa: Sörve)	57 54.2	57 54.6	- 0.4

Station	Meas. long.	Modern long.	Error
Öland: Böda	17° 05.2′	17° 04.7′	+ 0.5′
Gotland: Visby	18 24.2*	18 17.2	+ 7.0*
Fårö: Avanäs	19 23.9	19 21.0	+ 2.9
Gotska Sandön	19 13.1	19 13.2	- 0.1
Östra Bogskär [S of Åland]	20 22.9	20 25.2	- 2.3
Utö [SE of Åland]	21 21.5	21 22.1	- 0.6
Dagö: Dagerort (Hiiumaa: Köpu)	22 10.6	22 12.0	- 1.4
Ösel: Svarverort (Saaremaa: Sörve)	22 08.7	22 03.3	+ 5.4

tudes we first note a large error, denoted by an asterisk, for the station on Gotland. According to Schultén (1801) he had to leave the ship on the other side of the island and travel a long way on land to reach the station, which heavily disturbed the chronometer. This station, therefore, has been excluded from the further analysis. We now find, from the errors of the other stations, a standard deviation in longitude of 2.6′. This shows that the uncertainty in longitude determination with a chronometer on board a ship could easily be made smaller than with observing the Jupiter moons; see Table 4-2 in Chapter 4. The corrected positions of the islands in the Baltic Sea according to above were introduced into the successor to Nordenankar's sea atlas, the wide-spread sea atlas of the naval officer and hydrographer Gustaf af Klint.

It soon turned out that longitude determinations with ship chronometers were very useful in a much wider perspective. It so happens that London with the Greenwich observatory as well as all Nordic capitals with their national observatories are situated by the sea. Hence it would be possible to determine the *longitudes of the Nordic observatories relative to the Greenwich observatory using ship chronometers, in combination with star observations.* To reach a sufficient accuracy, expeditions had to be carried out by repeated voyages with a large number of chronometers. The introduction of steam-ships further improved the accuracy.

These large longitude expeditions were initiated by the German-Danish astronomer Heinrich Christian Schumacher, in cooperation with the Greenwich observatory. Schumacher (1827) writes:

”In the year 1824 the British Admiralty had a steam-ship fitted out and equipped with 28 chronometers, in order to perform the longitude connection between the Danish and English triangulations.”

More specifically the purpose was to determine the longitude difference between the observatories of Greenwich and Altona (close to Hamburg). The steam-ship made 6 voyages (3 forth and back) between Greenwich and Altona, via the island of Helgoland, with totally 34 chronometers on board (some chronometers being added by Schumacher). The chronometers brought Greenwich time, determined from the meridian passages of stars, to the observatory of Altona where it was compared with the local time, determined from stars there in the same way. The result was computed by Schumacher (1827) and found to be 9°56'38.6”. This can be shown to be in error by 7”; see Table 5-4. For the calculations he used a method specifically designed for this purpose by Carl Friedrich Gauss (1827), the German mathematician and astronomer, with whom Schumacher cooperated in several respects (see also Chapter 6).

At the same time Schumacher started a series of similar voyages to determine the longitude difference between the observatories of Altona and København (Rundetårn). These voyages were performed 14 times

Table 5-4. Successive longitude differences (in degrees, minutes and seconds) from Greenwich to the national observatories as determined by the great ship expeditions on the North Sea and the Baltic Sea in the 1820s, 1830s and 1840s, and their errors.

Observatories	Data source	Measured	Modern	Error
Greenwich – Altona	Schum. (1827)	9° 56′ 38.6″	9° 56′ 31.2″	7.4″
Altona – København	Schum. (1831)	2 38 17.2	2 38 07.5	9.7
København – Oslo	Hansteen (1849)	- 1 51 15.0	- 1 51 16.3	1.3
København – Stockholm	Schubert (1836)	5 29 11.8	5 28 50.7	21.1
Stockholm – Helsinki	Schubert (1836)	6 53 21.1	6 53 47.4	-26.3
Greenwich – Oslo	Summed	10 43 40.8	10 43 22.4	18.4
Greenwich – København	from	12 34 55.8	12 34 38.7	17.1
Greenwich – Stockholm	results	18 04 07.6	18 03 29.4	38.2
Greenwich – Helsinki	above	24 57 28.7	24 57 16.8	11.9
Greenwich – Altona	Struve (1846)	9 56 32.1	9 56 31.2	0.9
Altona – Pulkovo	Struve (1844)	20 23 07.8	20 23 08.3	- 0.5

during a period of several years, involving 10 chronometers. Most of the computations were performed by Schumacher's colleague, the Danish clockmaster and astronomer Peter Andreas Hansen (1831). The final result, however, was published by Schumacher (1831). The error here is close to 10″; see again Table 5-4.

A few years later, a Russian longitude expedition was carried out around the coasts of the Baltic Sea, headed by the naval officer T F Schubert (1836). It reached about 40 stations including several astronomical observatories, among them København, Stockholm and the recently erected one of Helsinki. At these the observatory astronomers were involved in the work with astronomically determining the local observatory times. The expedition comprised 3 voyages with up to 56 chronometers. As can be seen from Table 5-4 the errors for København – Stockholm and

Stockholm – Helsinki each exceed 20″ but, having opposite signs, almost cancel each other when added.

On the whole we may estimate the uncertainty of the longitude expeditions treated here at about 15″. This corresponds to 1 second in time.

The next decade witnessed the largest chronometer expedition in the world. It was carried out across the North and Baltic seas, between Greenwich and the new Russian central observatory at Pulkovo outside St. Petersburg. The Baltic part between Pulkovo and Altona, under the leadership of the German-Russian astronomer Wilhelm von Struve (1844), a former student of Schumacher, involved no less than 86 chronometers and 16 voyages. The North Sea part between Altona and Greenwich, under the combined leadership of Struve and his son, the Russian astronomer Otto von Struve (1846), involved 42 chronometers and 16 voyages. The results are shown in Table 5-4; they are extremely accurate (partly due to improved methods developed by Struve the elder). The errors turn out to be only 1″. This longitude connection to Greenwich would form the basis for further connections to Nordic observatories performed with new methods later on; see Chapter 6.

The Oslo observatory was being erected at this time. As soon as it had been completed, a chronometer expedition was arranged between the observatories of København and Oslo by two Norwegian scientists, the geophysicist Christopher Hansteen and the astronomer Carl Fearnley (1849). Voyages were performed 14 times, involving 21 chronometers; also this result is given in Table 5-4.

A consequence of all these ship chronometer expeditions was that longitudes now became better established relative to Greenwich than to Paris. This was contrary to the case during the era of the Jupiter moons. Consequently, the nautical charts in the wide-spread Klint's sea atlas introduced longitudes relative to *Greenwich* in 1849. In Table 5-4 we have not only included the longitude differences between the national observatories and their errors, but also the longitudes of the observatories relative to Greenwich. For the sake of completeness we have also, in Table 5-2, collected the latitudes of the same observatories determined during this time; we will return to these data in the following chapter.

Finally it should be noted that several of the errors in Tables 5-2 and 5-4 are so small that the effects of the so-called deflections of the vertical become important here. For a discussion of this and the modern coordinates in these tables, see Chapters 7 and 8 (especially Section 8.1).

6. Stars and triangles on continents: Topographic maps

6.1 The Earth as an ellipsoid of revolution

After Maupertuis' demonstration of the flattening of the Earth at the poles, a large number of further arc measurements confirmed the Earth's flattening. It became clear that the general shape of the Earth is that of an *ellipsoid of revolution,* with a *semi-major axis a* equal to the radius of the equator, and a *semi-minor axis b* equal to the distance along the rotational axis from the centre to the pole. Thus, at the pole a part of the Earth corresponding to $a - b$ is "missing", in comparison with a spherical Earth of radius a. The relation of the missing part to the whole radius is known as the *flattening f* of the Earth, $f = (a - b)/a$. Alternatively the *eccentricity e* of the ellipsoid is used, $e^2 = (a^2 - b^2)/a^2$. Approximate modern numerical values of the above parameters are a = 6 378 km, b = 6 357 km, f = 1/298, e^2 = 0.00669.

A spherical Earth has a constant radius. On an ellipsoidal Earth the concept of radius is no longer unique. Imagine an arc on the surface of the ellipsoid, created as the intersection between the ellipsoid and a normal plane. A sufficiently short part of this arc may be approximated by a part of a circle. This circle has a certain radius, called the radius of curvature of the arc, but this radius depends on the location of the arc on the ellipsoid as well as on its direction. The radius of curvature in the direction of a meridian is known as the *meridional radius of curvature, M.* It is smallest at the equator and largest at the pole; it is a function of latitude φ according to

$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}} \qquad (6\text{-}1)$$

The radius of curvature in the direction perpendicular to the meridian is sometimes known as the *perpendicular radius of curvature, N*. It is a function of latitude φ according to

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}} \tag{6-2}$$

These formulae were used by Maupertuis (1738) and derived by him five years earlier. Approximate numerical values at the equator and the pole are M_e = 6 335 km and M_p = 6 400 km along the meridian, and N_e = 6 378 km and N_p = 6 400 km perpendicular to that.

The length of a *short arc along a meridian* can now be written

$$\Delta s_m = M \, \Delta\varphi \tag{6-3}$$

Correspondingly the length of an *arc along a parallel circle* can be written

$$\Delta s_p = N \cos\varphi \; \Delta\lambda \tag{6-4}$$

Using the ellipsoidal parameters of Maupertuis and applying the above formulae, Celsius and his assistant Olof Hiorter (1741) published the first tables of the length of a degree for various latitudes on the Earth. (Hiorter also was married to Celsius' sister.) It is illustrative here to study the length of a degree or a minute of arc along a meridian. Inserting the equatorial and polar values of M given above into (6-3) yields 1′ = 1 843 m at the equator and 1′ = 1861 m at the pole. Thus a short meridian arc of a fixed latitude difference is 1 % longer at the pole than at the equator. The average of the two values, 1 852 m, is the definition of a *nautical mile.*

The length of a much *longer arc along the meridian* requires integration; an arc from the equator to an arbitrary latitude becomes

$$s_m = \int_0^{\varphi} M(\varphi) \, d\varphi \tag{6-5}$$

This integral leads to intricate problems and series expansions typical for calculations over long distances on the ellipsoid; we leave them aside here. The original definition of a *metre,* introduced in France in 1798, is based on (6-5) taken from the equator to the pole, one metre being defined as 1 / 10 000 000 of that distance.

When computing a triangulation over a part of the Earth it becomes necessary to take the curved shape of the Earth into account. To begin with, this has to be done when handling the triangles in the network, i.e. when calculating the lengths of their sides from the measured angles and the measured baseline. In this case a local spherical approximation of the ellipsoid is sufficient; the calculations could be done by applying spherical trigonometry.

After trigonometry has given the sides of the triangles, the differences of latitude and longitude between the stations within the network may be found out. If a length of a side is known, together with its azimuth, the components of the side along the meridian and the parallel, Δs_m and Δs_p, can be calculated. Putting these quantities into equations (6-3) and (6-4) above, and solving for $\Delta\varphi$ and $\Delta\lambda$, one obtains the latitude and longitude differences in the network. Taking the latitude and longitude at an astronomical station into account, the latitudes and longitudes of all stations in the triangulation network can be determined.

Let us summarize the c*alculations of positions through triangulation* in four steps.

A. Choose a well-determined ellipsoid (defined by its semi-major axis and its flattening or eccentricity) on which to perform the calculations.

B. Calculate the sides of the triangles of the network using trigonometry and a local spherical approximation of the ellipsoid.

C. Calculate the differences in latitude and longitude between the stations in the network using (6-3) and (6-4), the radii of curvature of the ellipsoid being given by (6-1) and (6-2).

D. Calculate the final latitudes and longitudes of the stations in the network by adding the latitude and longitude of a fundamental astronomical station.

A main consequence of the above procedure is that the latitudes and longitudes resulting from a triangulation become dependent on the choice of the ellipsoid. A different choice of the values of the defining parameters of the ellipsoid will give different values of the radii of curvature and, in the end, different latitudes and longitudes.

We noted earlier that a short meridian arc of a given latitude difference is 1 % longer at the pole than at the equator. Thus a degree is about 1 000 m longer at the pole than at the equator. This can be compared with the uncertainty of the Åland triangulation in Chapter 5 which was shown to be about 100 m. Triangulation for mapping, therefore, already from its break-through required an ellipsoid for its computation.

Let us finally take a closer look at equation (6-3). This simple equation contains the fundamental possibilities for three different applications, all of which have been dealt with in this book:

1. *Arc measurements for the figure of the Earth:* Δs and $\Delta\varphi$ are measured, M is solved for. Doing this for two arcs allows the use of (6-1) to solve also for a and e.

2. *Astronomical positionings for mapping:* $\Delta\varphi$ is measured and M is known from (6-1), Δs is solved for.

3. *Triangulation for mapping:* Δs is measured and M is known from (6-1), $\Delta\varphi$ is solved for.

Items 2 and 3 may also be formulated applying (6-4) and (6-2).

6.2 Triangulation inland and the first topographic maps

At the beginning of the 1800s the Napoleonic Wars changed the scene for mapping. It became apparent that accurate *topographic maps over whole countries* were required for military purposes. This caused a need for *nation-wide triangulations of scientific quality,* and lead to the military taking over the official mapping. Soon it became apparent that such maps also were useful for many civilian purposes in society.

The calculations of the national triangulations required a well-determined Earth ellipsoid. In the Nordic countries as in several other countries, therefore, the triangulations in one way or the other were connected with renewed arc measurements for improving the knowledge of the ellipsoid.

As in the foregoing chapter it started with an arc measurement at the Arctic circle. The result from Maupertuis' expedition there had turned out to deviate too much from later arc measurements performed on other parts of the Earth. Therefore, a renewed arc measurement at the Arctic circle was performed by the Swedish mathematician Jöns Svanberg in 1801 – 1803, born in the area. He adopted the methods that had recently been used in France for the special arc measurement there for defining a new unit of length, the metre. When presenting the results, Svanberg (1805) also had adopted the unit of length itself and counted in metres, two generations before the international break-through for the metre.

The same year as Svanberg's book was published a military geodetic institute, mostly known as the Royal Topographic Corps, was created in Sweden, with the aim of performing triangulations and constructing topographic maps over the whole country. Svanberg was appointed its first scientific leader. Triangulating Sweden is not easy because of its vast uninhabited forests; they are difficult to walk through with scientific equipment and they prevent sights between stations. As a result, the triangulation of Sweden lasted for nearly a century; it was carried out in 1815 – 1890. Some parts of the country could not be covered by triangles and the mapping there had to rely on less accurate positioning.

The fundamental astronomical station of the Swedish triangulation was, as earlier, the Stockholm observatory. Its latitude was redetermined by Svanberg's successor, the astronomer and geodesist Simon Cronstrand (1811), using a relatively small number of star observations. His value differed very little from the old one of Wargentin (1759) as recalculated by Cronstrand with a new star catalogue, and the value adopted for the triangulation was the average of the two, 59°20'34.8"; see Table 5-2 (Chapter 5). As can be seen there, the error in both Wargentin's (1759) value and Cronstrand's (1811) value is 2", although of opposite signs. The longitude of the observatory was as usual put to 0°, keeping the zero meridian at Stockholm. Its connection to the Greenwich meridian is treated in Section 5-4; see also the end of this section. The ellipsoid adopted for the triangulation was an improved version of the one that Svanberg (1805) had computed from his own arc measurement combined with three others.

A few decades later Cronstrand's successor, the Swedish geodesist Haqvin Selander (1835), made a complete redetermination of the latitude of the Stockholm observatory, using 108 star observations. From Table 5-2 we find that his error is only 1". Nevertheless, the earlier value continued to be used for the triangulation.

When the southern half of the triangulation had been completed after half a century, including the time-consuming calculations, the resultant coordinates were made public by Selander (1866), after much hesitation. The reasons given for the hesitation throw light on a classical and permanent problem with official coordinates:

"The following results of determinations of positions in Sweden by the Topographic Corps have already since a long time been calculated and been meant for printing. The printing has, however, been postponed year after year to await the finishing of certain control measurements, like a baseline measurement, astronomical determinations of latitudes and azimuths, connections to the Norwegian, Danish and Russian triangulation networks etc. These reasons called for a complete recalculation which … cannot be avoided if the work shall provide a solid foundation for all time for maps of Sweden. The control measurements were performed. The recalculation started and had progressed quite far. Then the Mid-European arc measurement cropped up which will have a considerable impact on all positions …"

The coordinates were published with the warning that they could be expected to undergo changes in the future.

The accuracy of the coordinates can be judged from certain investigations made by Selander's successor, the Swedish geodesist Per Rosén (1879). We will revert to this problem towards the end of this section.

While the Swedes in the beginning had turned to the French for scientific inspiration, the Danes turned to the Germans. In 1816 a national geodetic institute, the Danish Arc Measurement, was founded, with the German-Danish astronomer Heinrich Christian Schumacher as its first scientific leader. He had been a student of Gauss. Schumacher now arranged a cooperation with Gauss: While Schumacher was making a combined arc measurement and triangulation of Denmark, Gauss would extend this work into the adjoining state of Hannover, south of Denmark.

The first results of the Hannoverian triangulation were published by Gauss (1828); he personally made all the numerical calculations. Here and in other related works Gauss applied several of his recent mathematical ideas. He applied the method of least squares to deal with overdeterminations, he tested the theory of curved surfaces on ellipsoidal triangles, and he introduced a new map projection of his, later on used all over the world.

The triangulation of Denmark was carried out, with interruptions, in 1816–1870, partly benefiting from the work of Gauss. The fundamental astronomical station of the triangulation was, as earlier, the København observatory (Rundetårn). Its latitude was redetermined by Schumacher (1827a) using no less than 279 star observations. His value adopted for the triangulation was 55°40′52.6″, differing slightly from the old one of Bugge (1779); see Table 5-2. From there we find that, while the error in Bugge's (1779) value is 4″, the error in Schumacher's (1827a) value is only 1″. The longitude of the observatory was as usual put to 0°, keeping the zero meridian at København. Its connection to the Greenwich meridian is treated in Section 5.4. The ellipsoid adopted for the triangulation was a modification of the one by Walbeck (1819), treated below; the Walbeck ellipsoid had been used by Gauss.

We should note here that with the latitude determinations of Schumacher (1827a) and Selander (1835), as well as with the longitude determinations of Struve (1844, 1846) in Section 5.4, the error in fundamental astronomical positioning had now reached below 1″. This error is so small that the effect of the so-called deflection of the vertical becomes important here; for a discussion of this and the modern latitude and longitude values in Tables 5-2 and 5-4, see Chapters 7 and 8 (especially Section 8.1).

The results of the Danish triangulation were presented in three comprehensive volumes by Schumacher's successor, the Danish military geodesist (and minister of finance) Carl Georg Andræ (1867, 1872, 1878), followed by a fourth volume by Andræ and the German astronomer Christian Albert Friedrich Peters (1884). In the last volume, when looking back on the now completed work, Andræ & Peters (1884) make a reflection probably common to all who have experience from this kind of scientific project lasting for several generations:

"Measurements that, with various interruptions, have covered a time span of almost 70 years, in many respects cannot fulfil the scientific demands that nowadays might be considered fully justified. … In addition to this, the original plan, irrespective of how satisfactory it might have appeared at that time, nowadays will require a lot of modifications and additions."

Thus, at the completion of the work, one could already foresee the need for further improvements.

For Norway the political landscape had changed at the end of the Napoleonic wars. Norway had been separated from Denmark and turned into a country of its own, in a loose union with Sweden. This meant that the Geographical Survey of Norway now was allowed to handle the fundamental positioning for mapping on its own.

Triangulating Norway is quite a special task because of its extensive uninhabited mountain areas; the mountains are not always easy to climb with scientific instruments. It took some time before the triangulation could start; it was carried out in 1826 – 1875. In northernmost Norway use could be made of Struve's arc measurement; see below. It extended all the way up to the Arctic Sea, its end point being at Hammerfest (Figure 6-1).

The fundamental astronomical station of the Norwegian triangulation was, to start with, still the flagpole at the Kongsvinger fortress. However, it was soon replaced by the observatory of Oslo (at that time named Christiania), founded by the Norwegian geophysicist Christopher Hansteen, scientific leader of the triangulation. The latitude of the Oslo observatory was determined by Hansteen and the Norwegian astronomer

Figure 6-1. Fuglenes near Hammerfest at the Arctic Ocean, northern end point in 1852 of the Russian-Scandinavian arc measurement, led by Struve (photo B G Harsson).

Carl Fearnley (1849). Their result adopted for the triangulation was 59°54′43.7″; see Table 5-2. The longitude was put to 0°, thereby making the Oslo observatory the new zero meridian of Norway. Its connection to Greenwich is treated in Section 5-4. The ellipsoid chosen for the triangulation was the one by Bessel (1841); see next section.

The resultant coordinates of the triangulation were not published in full. A summarizing investigation was made by the Norwegian astronomer Hans Geelmuyden (1895); we will return to that in Chapter 7.

Also for Finland the political landscape had changed during the Napoleonic wars. Finland, together with Åland, had been separated from Sweden and turned into an autonomous part of the Russian empire. This meant that the fundamental positioning for the mapping of Finland came into the hands of Russia.

The Russian authorities did not perform a national triangulation of Finland. Instead the scientists at the newly founded observatory of Pulkovo outside St. Petersburg contributed with a triangulation chain through Finland. This was a part of a very long arc measurement, starting in present Ukraine, crossing the whole of Finland, touching northeastern Sweden and extending into northernmost Norway. The work was performed by Struve the elder (introduced in Chapter 5), partly together with Tenner, Hansteen and Selander (1857 & 1860). Their arc was used as a basic triangulation in Finland and also in the aforementioned parts of Sweden and Norway. Later connected to this triangulation was the observatory of Helsinki, founded and astronomically positioned by the German-Finnish astronomer Friedrich Argelander (1837); see Table 5-2.

An ellipsoid used in Finland and Russia was computed by the Finnish astronomer Henric Johan Walbeck (1819), applying for the first time in this context the method of least squares. Walbeck applied this new method to find the most likely ellipsoid parameters from six arc measurements on different parts of the Earth, one of them being the renewed Swedish one by Svanberg. Walbeck's ellipsoid was later used by Gauss for his triangulation calculations and his map projection.

The triangulations in the end resulted in more or less complete series of *topographic maps* covering the respective country. Also other map series, like *land use maps* and *nautical charts,* were produced with the triangulations as their basis. In principle the basic triangulation was densified by local triangulations, which in their turn were densified by even more local measurements of directions and distances. In this way all objects could be placed on the map with reasonably correct coordinates and, hence, the whole map or chart constructed. An example of a topographic map based on these triangulations is shown in Figure 6-2.

Nautical charts had long since been made in the *Mercator projection*, a conformal cylindric projection suitable for navigation. For the topographic maps some other conformal projection was needed. A *conformal projection* preserves angles and, thereby, the shapes of small objects like islands, lakes etc. in the map, whereas the sizes of the objects inevitably become erroneous. A new such map projection with less erroneous sizes had been developed by the German mathematician Johann Heinrich

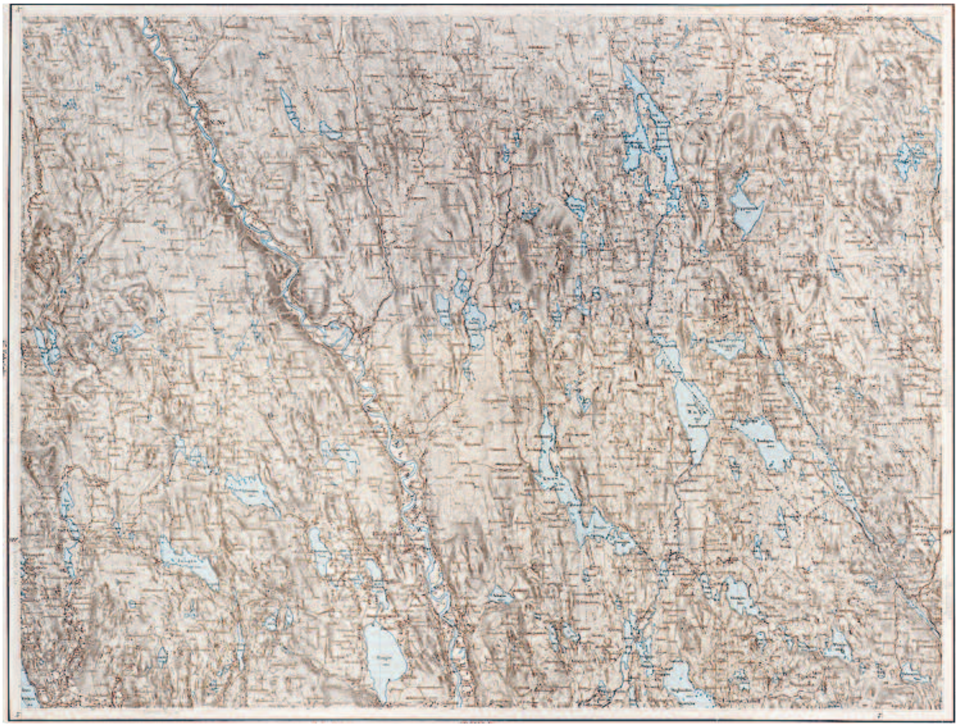

Figure 6-2. Topographic map (with a part of the river Klarälven) of the kind issued in the second half of the 1800s, based on triangulation.

Lambert (1772), a conformal conic projection. *Lambert's projection* was now introduced for most *topographic maps*, in Sweden in a special version constructed by the military geodesist Carl Gustaf Spens (1817), minimizing the projection errors.

The different kinds of measurements performed in a triangulation network provide a possibility of investigating the accuracy of the network. This can then be used for roughly estimating the accuracy of the resultant coordinates. The baseline length in a triangulation network is considerably more accurate than the angles in the network. Hence an estimate of the accuracy of the network can be obtained by measuring several baselines in different parts of the network, and then calculating the length of a baseline starting from another baseline, using all the angles in between. Comparing the calculated length of the baseline with the measured length gives an indication of the accuracy of the network.

Doing so both for the Swedish network and for adjacent parts of the Danish and the Norwegian networks, Rosén (1879) found an uncertainty of the order of 1 : 20 000.

In Chapter 5 we estimated the corresponding quantity for the pioneering Åland triangulation at 1 : 2 000. Hence the triangulation of the 1800s should be one order of magnitude better than the best one of the 1700s. As the uncertainty of the coordinates of the Åland triangulation was found to be about 100 m, we may conclude that the uncertainty of the present triangulations should be of the order of 10 m.

We have seen that each country had selected a national zero meridian for its triangulation. For international cooperation and for nautical charts, however, it was necessary to know the relation between the national zero meridian and the international one through Greenwich. This had been achieved through the great chronometer expeditions between the Greenwich observatory and the Nordic observatories during the first half of the 1800s, as described in Chapter 5. Now, in the second half of the 1800s a new method of transferring time from one place to another was introduced. Time information could be sent immediately via telegraph (later on via radio). This gave rise to a series of more accurate longitude determinations between the observatories.

A telegraphic longitude determination between København and Altona was performed by Peters (1884). The connection between Altona and Greenwich still rested on the improved chronometer expedition by Struve & Struve (1846). In a Swedish-Norwegian-Danish cooperation the astronomers Georg Lindhagen, Carl Fearnley and Frederik Christian Schjellerup (1890) performed telegraphic longitude determinations between København, Oslo and Stockholm. (Lindhagen also was married to a daughter of Struve the elder, or a sister of Struve the younger.)

Although measured last, the telegraphic longitude determination between Stockholm and Helsinki and further to Pulkovo was published first, by the Russian and Swedish-Russian astronomers V Fuss and Magnus Nyrén (1871). From there the longitude relative to Greenwich had been fixed by the world´s largest chronometer expedition, that between Pulkovo and Altona by Struve (1844) combined with the one between Altona and Greenwich by Struve & Struve (1846) mentioned above; see

Table 6-1. Longitudes relative to Greenwich (in degrees, minutes and seconds) of the national observatories as determined in the late 1800s, and their errors.

Observatory	Data source	Measured	Modern	Error
Oslo	Lindhagen (1890)	10° 43′ 23.3″	10° 43′ 22.4″	0.9″
København	Peters (1884)	12 34 39.6	12 34 38.7	0.9
Stockholm	Fuss (1871)	18 03 29.8	18 03 29.4	0.4
Helsinki	Fuss (1871)	24 57 17.2	24 57 16.8	0.4

Section 5.4. This had the consequence that the Swedish connection to Greenwich became defined via Pulkovo.

The longitude results for the national observatories are collected in Table 6-1. The modern longitudes there are taken from Chapter 8 (Section 8.1).

6.3 Continental triangulations and maps

As we have seen above, triangulations and official mapping were long-lasting national projects where each country strived to use the best Earth ellipsoid available at the time of starting the computations. This lead to different countries choosing different ellipsoids, causing the final coordinates to be inconsistent across the national borders. We have also seen that different countries established different astronomical stations as their respective starting points, known as datum points, for calculating coordinates. Each country thus had its own *reference system*, in which the coordinates were given, based on a specific *ellipsoid* and a specific *datum point*.

To overcome this, international cooperation became necessary. In 1861 an international organization for this purpose was founded, soon to be known as the International Earth Measurement and later as the International Association of Geodesy. Its central person was the German geodesist and geophysicist Friedrich Robert Helmert. Helmert (1886) started to connect national triangulations with each other, recalculating them on a common ellipsoid, starting from a common datum point. The common datum point was Berlin. The common ellipsoid was the one by the German mathematician and astronomer Friedrich Wilhelm Bessel (1837 & 1841). Thereby the Bessel ellipsoid achieved the status of an unofficial international standard. This ellipsoid was based on ten arc measurements from different parts of the world, including the renewed Swedish one by Svanberg, the Danish-Hannoverian one by Schumacher and Gauss, and the southern part of the Russian-Scandinavian one by Struve.

During the first half of the 1900s the Nordic countries performed renewed and more dense national triangulations as a foundation for more accurate topographic and other maps. In this connection a greatly improved method of measuring the lengths of baselines, using thin wires instead of rods, had been introduced by the Swedish geodesist Edvard Jäderin (1915), in an Arctic arc measurement on Spitsbergen (Svalbard); it soon became a standard procedure in triangulation.

For the computations of their triangulations, Norway and Sweden now adopted the international Bessel ellipsoid, Norway, however, a modified version of it (due to a differing conversion to the metre as a unit of length). Sweden also abandoned its national datum point (at least as far as latitudes are concerned) and started its calculations from the Danish datum point in København. Denmark and Finland, however, starting slightly later, adopted the new ellipsoid of the American geodesist John Hayford (1909 & 1910), at that time replacing that of Bessel as an international ellipsoid. Finland, now making their first national triangulation, introduced the observatory of Helsinki as their datum point. Thus also the renewed triangulations were calculated in separate national reference systems, although the Swedish one via København had certain connections to the old Central European system initiated by Helmert. This Swedish system, in a slightly modernized version, is still partly in use although it has really old roots: It ultimately rests on Schumacher's latitude determination of København in 1820 – 1821, Struve's longitude expedition to Greenwich in 1843 – 1844 and on Bessel´s ellipsoid based on measurements dating back to 1735!

After the First World War the countries around the Baltic Sea in 1924 formed the so-called Baltic Geodetic Commission, with the main task of connecting the triangulations around the Baltic proper and recalculating them in a common reference system. This work was completed by the Finnish geodesist Victor Ölander (1949), using the Hayford ellipsoid and a new way of defining the datum point (see Chapter 7).

After the Second World War nearly all national triangulations in Europe were brought together, on an American initiative, by the International Association of Geodesy (IAG) and recalculated with the newly invented first generation of computers. In this way the American geodesist Charles Whitten (1952) managed to present a common reference system for a large part of Europe, known as the European Datum (ED). This was based on the Hayford ellipsoid with the datum point at Potsdam close to Berlin. The ED coordinates soon became widely used in European cooperation. Moreover, they were adopted as national coordinates in both Denmark and Norway and used there for their topographic maps.

The Nordic triangulations during the first half of the 1900s in the end resulted in new complete series of topographic and land use maps as well as nautical charts, this time with the additional aid of aerial photography. An example of a topographic map based on these renewed triangulations is shown in Figure 6-3.

When the above triangulations were about to begin, the German geodesist Louis Krüger (1912) had presented explicit and accurate formulae for the favourable map projection invented by Gauss nearly one century earlier. Gauss' projection is a conformal cylindric projection, a relative of the Mercator projection, but with the cylinder being tangent to the Earth along a meridian instead of along the equator. This makes the cylinder of Gauss an elliptic one instead of a circular one, leading to mathematics resulting in long series expansions. With Krüger's explicit formulae available, most countries, including the Nordic ones, now introduced the *Gauss projection* (transverse Mercator projection) for the *topographic and other accurate land maps*. In connection with issuing the new maps Greenwich was introduced as zero meridian also on the land maps, not only on the marine charts as before.

Figure 6-3. Topographic map (with a part of the river Klarälven) of the kind issued in the second half of the 1900s, based on renewed triangulation (combined with aerial photography).

The internal uncertainty of the resultant coordinates of the renewed triangulations during the first half of the 1900s can be estimated at about 2 m, based on comparisons with a later and especially accurate triangulation (Section 7.2). This is five times better than the corresponding uncertainty of the coordinates stemming from the triangulations during the 1800s as found in Section 6.2; see Table 6-2.

Table 6-2. Estimated internal uncertainties (in m) of the coordinates of the national mapping triangulations during three centuries.

Century	Uncertainty
1700s	> 100 (100 – 500)
1800s	10
1900s, first half	2
1900s, second half	0.1

Calculating a large triangulation network was a tremendous work; it could not be performed in one piece, but had to be split up into several blocs which afterwards were in some way tied together. When the first Nordic *computers* had been constructed in the early 1950s they were right from the beginning used for geodetic applications. The Danish geodesists Torben Krarup and Bjarner Svejgaard (1956) started developing mathematical methods suitable for solving large geodetic problems with computers. They even constructed a computer themselves, very much used for this kind of work.

After having completed his calculations of the European triangulations Whitten (1952) writes:

”With the experience which has been gained in the completion of these two projects [the northern and south-western blocs] and the rapid development which has taken place in high-speed computing equipment during the past few years it is now practical to at least think of adjusting all the triangulation of a continent into one homogenous network. I believe that future generations will go beyond the limitation of continents and devise methods for the adjustment of a world network of triangulation.”

However, in order to go ”beyond the limitation of continents” one would have to face a remarkable problem. This problem did not have to do with the surface of the Earth, nor with the stars in the sky, but, surprisingly enough, with the unknown interior of the Earth.

7. Stars and satellites: Mapping in a global system

7.1 Star observations and hidden masses inside the Earth

While the first national triangulations were going on and were being analyzed in several countries on the European continent, a suspicion began to grow. Maybe the vertical, i.e. the plumb line, did not everywhere coincide with the normal to the Earth ellipsoid? Not only visible mountains but also possible irregularities in the mass distribution inside the Earth would cause such an effect. It was already known that density within the Earth's crust was not the same all over, and it was also known that density increased towards the centre of the Earth. Based on this Gauss (1828) introduced a new fundamental surface of the Earth: the equipotential surface of the Earth's gravity field coinciding with mean sea level, i.e. a surface everywhere perpendicular to the plumb line. This surface was later termed the *geoid*. The geoid would deviate in an unknown way from the ellipsoid because of the irregular mass distribution within the Earth.

The British mathematician and physicist George Gabriel Stokes (1849) derived a formula showing how this deviation was dependent on anomalies in the gravity field of the Earth. However, the formula required not only that gravity could be measured with sufficient accuracy, but also that it was measured all over the globe. To compute the deviation of the geoid from the ellipsoid at a certain point it was necessary to integrate the gravity anomalies over the Earth as a whole. Because of this the formula of Stokes could not be applied in practice (until our own days).

However, the problem needed to be investigated in some way since it would affect the astronomical determination of latitudes and longitudes. When putting up an astronomical instrument for positioning it is adjusted with a spirit level. The spirit level "feels" the direction of the plumb line, or the vertical. If the direction of the vertical deviates from the normal to the ellipsoid, this deviation will directly affect the measured latitude and longitude. Since the error caused by this phenomenon

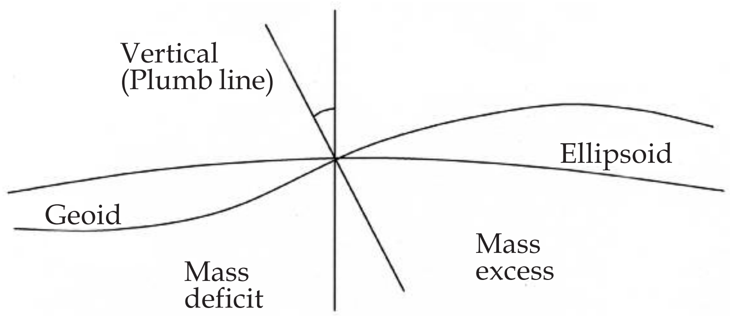

Figure 7-1. Deflection of the vertical due to irregular mass distribution inside the Earth.

is due to hidden masses inside the Earth (see Figure 7-1) we have a very tricky situation: The origin of the problem is invisible!

Now, the question could be partly investigated by making astronomical determinations of latitude and longitude at several stations in different parts of a triangulation network, and then comparing the astronomical coordinates with the triangulated ones. The astronomical measurements are directly dependent on the direction of the vertical. The measurements of the lengths and horizontal angles in the triangulation network are not. Hence the astronomical coordinates will differ from the triangulated ones, if there are local or regional deflections of the vertical from the normal to the ellipsoid. Denoting the *astronomical latitude and longitude* by Φ and Λ, and the *triangulated (geodetic) latitude and longitude* by φ and λ, we may write

$$\Phi = \varphi + \xi \tag{7-1}$$
$$\Lambda = \lambda + \eta / \cos \varphi \tag{7-2}$$

Here ξ and η are the *deflections of the vertical* in the south-north and west-east directions, respectively.

The search for deflections of the vertical was initiated in Germany by Helmert. He not only strived to connect national triangulation networks to each other but also to perform accurate astronomical positioning on selected stations within the networks, thereby allowing discoveries of deflections of the vertical. Helmert (1880) writes:

”The calculation of the deflections of the vertical for assumed dimensions of the reference ellipsoid presupposes that for one point the deflection of the vertical either is put to zero or is included as an unknown quantity in the calculation. Hereby we naturally suppose that all triangulation networks are connected with each other.”

After several years’ work Helmert (1886) could publish the first values of deflections of the vertical for a number of German states, mainly along the Baltic Sea. Arbitrarily putting the deflections at Berlin to zero (on the Bessel ellipsoid) he found that the deflections at other stations amounted to $\xi \approx \eta \approx 10''$. This is one or two orders of magnitude larger than the uncertainty of the astronomical positioning of an observatory at that time (Sections 6.2 and 7.4). Two decades later Helmert’s co-worker A Börsch (1906) had extended the area of determined deflections to include also Denmark.

In Scandinavia Rosén (1889) made the first determinations of deflections of the vertical by making astronomical latitude determinations on triangulation stations in Sweden and northern Norway. Putting the deflection at Stockholm to zero, he obtained deflections at the other stations of more or less the same magnitude as Helmert. Geelmuyden (1895) added deflections in southern Norway.

In Finland, one generation later, the gravimetric method was tried. Using a rather limited number of gravity anomalies determined there, Ölander (1931) estimated the deflections of the vertical. These are nothing but the inclinations of the geoid, which thus can be found by differentiating Stokes’ formula. Because of the lack of a more global coverage of gravity data neither this attempt could result in any absolute values of the deflections.

When calculating the European triangulations in the common ED system, it became possible to calculate also deflections of the vertical in this system, based on the Hayford ellipsoid. This work was initiated by the German geodesist Helmut Wolf (1949), starting from Potsdam close to Berlin. But still all deflections rested on some more or less arbitrary assumption of the values of the deflections at this starting point (sometimes these values were chosen to minimize the deflections within a certain area).

Let us look back a little to put the problem of deflections of the vertical into perspective. In the early days, when astronomical positioning was performed with accuracies of the orders of a degree or a minute (Chapters 2 and 3, partly also 4), all positions could be regarded as being given in one and the same system, a global system. Later on, the accuracy of the astronomically determined positions increased to the order of a second (Chapter 6, partly also 5). It then turned out that these coordinates were disturbed by deflections of the vertical being considerably larger than the uncertainty of the positioning. This meant that the coordinates of a triangulation founded on a certain astronomical station could be regarded as belonging to one system, but that the coordinates of another triangulation founded on another astronomical station must be regarded as belonging to another system. The relation between the two systems remained unknown as long as the deflections of the vertical at the two astronomical stations were not known, or the two triangulation networks were not connected.

As we have seen, however, deflections of the vertical could not be determined in an absolute sense, only relative to a neighbour station, and triangulation networks could not be connected over longer distances than those allowing sights between them. This made it *impossible to connect different continents with each other.* Triangulation between continents obviously was prevented by the observer not being able to see across the ocean. Neither could such a global method as astronomical positioning be used for bridging the gap between continents, since the astronomically determined positions did not come out in a consistent system because of the deflections of the vertical. And these deflections are caused by *invisible masses*! As the deflections of the vertical could be expected to reach 10″ – 20″ or one quarter of a minute of arc, the resultant errors in the latitudes and longitudes would correspond to up to 500 m on the surface of the Earth.

To summarize the problem:

1. Triangulation between continents is prevented by lack of sight across the ocean.

2. Astronomical positionings on different continents are not comparable to each other because of perturbations by hidden masses inside the Earth.

7.2 How to connect continents?

In 1945 a total solar eclipse occurred, visible both in the Nordic countries and in Canada. The Finnish geodesist Ilmari Bonsdorff (1944) proposed to use this and similar events for connecting Europe and America across the Atlantic. The idea was the following.

The solar eclipse does not occur at the same instant all along the path on the Earth's surface from where it is visible. The times of the beginning and the end of the eclipse are dependent on the motion of the moon and, thereby, on the location on the Earth from where the eclipse is observed. Consequently, if the times of the totality of the eclipse are accurately measured at two different locations on the Earth, one on each side of the Atlantic, the distance between the locations might be deduced. The observations showed that it was not easy to measure these times accurately enough; moreover, total solar eclipses are both rare and geographically limited events.

The same year marked the end of the Second world war. Just a few months after the war had ended a cooperation took place between the British Royal Air Force, American military surveyors, and Danish and Norwegian geodesists, for connecting the triangulations of Denmark and Norway across the sea of Skagerrak. This was a kind of three-dimensional triangulation using parachute flares dropped from aeroplanes. These flares were observed simultaneously from three triangulation stations in Denmark and three in Norway. The experiment was quite successful, although hastily prepared, according to the results calculated by the Danish geodesist Ove Simonsen (1949). However, it was a matter of bridging a limited sea area, not an ocean.

A more far-reaching idea was presented by the Finnish physicist and astronomer Yrjö Väisälä (1946). His idea was the following.

Imagine a rocket sent up high enough above the Earth's surface. During night-time this rocket can be observed (photographed) against a background of stars on the celestial sphere. The observed position of the rocket against this background will be different as seen from different observation points on the Earth. Consequently, if the positions of the rocket on the celestial sphere, expressed as declination and right ascension, are determined simultaneously from several observation points on two continents separated by an ocean, the relative positions of the points on the Earth could be determined. These positions on the Earth would then be in a common system and, if extended to span the whole Earth, independent of any deflections of the vertical.

Remarkably enough, Väisälä (1946) considers a further development of this idea:

”If rocket missiles can be developed to such a degree that it would be possible to realize small moons which would circle the earth at an altitude of some thousands of kilometers with a period of only several hours, we should obtain practically eternal light sources for a giant triangulation and these light sources could also be used for physical measurements of the earth. A simple calculation reveals that an artificial moon several decimeters in diameter could be followed with medium-sized apparatus.”

What Väisälä proposes here is a kind of satellite triangulation. Later he and his colleague Liisi Oterma (1960) developed the method for practical use, but when Väisälä made his proposal man-made satellites, ”artificial moons”, did not yet exist in reality!

At about the same time as the above works were going on, the Swedish geodesist Erik Bergstrand (1948) invented a new method of measuring distances, using the emission of electro-magnetic waves in the form of light. The principle is simple. The velocity of light is a known constant. Hence, from the time it takes to send a light signal from one station to another it is possible to find the distance between the two stations. Bergstrand developed this method so that it could be used in practise with an accuracy of, at its best, 1 : 1 000 000. He also turned it the other way around: An internationally widely used value of the velocity of light was determined by Bergstrand (1950). Positioning problems thus inspired him to find the velocity of light just as it had inspired Rømer 300 years ago (Section 4.1).

Bergstrand's invention led to Sweden performing a third triangulation, now measuring distances between all the stations instead of angles. In this respect the triangulation mentioned is probably a unique one. The internal uncertainty of the resultant coordinates is about 0.1 m (Reit, 1995), one order of magnitude smaller than that of the last traditional triangulation; see Section 6.3 (Table 6-2).

Now, the optimal way of trying to connect continents would be to combine the works of Väisälä (1946) and Bergstrand (1948):

1. To connect continents and get rid of the unknown deflections of the vertical, use *artificial satellites,* as suggested by Väisälä.

2. To make the connections accurate enough, measure *distances* to the satellites using waves travelling with the velocity of light, in accordance with the principle of Bergstrand.

A combination of this kind actually forms part of the basis of today's satellite positioning, developed in America. However, before discussing satellites we need to make a visit to the most distant parts of the universe.

7.3 Distant galaxies and close satellites

In 1964 a high quality *radio telescope* was built at the *Onsala* space observatory south of Göteborg (see Figure 7-2), founded fifteen years earlier by the Swedish physicist Olof Rydbeck. Here radio waves from some recently discovered radio sources in the universe, known as quasars, could be received and analysed. It had just been found out that the quasars were a kind of *extremely distant galaxies (star systems);* in fact, they were the most distant objects observed in the whole universe. Their distance exceeded 10 billion light years, implying that they were observed in a state representing the childhood of the universe.

In comparison with the size of the Earth, which is of the order of 1/10 of a light second, the distance to the quasars may be considered as infinitely large. Because of that, radio signals from quasars received at two widely separated observatories on the Earth can be considered as parallel to each other. This opens up the possibility to compare the pha-

Figure 7-2. Onsala space observatory, receiving radio waves from distant star systems, a fundamental station for VLBI and satellite positioning (photo B Hansson).

ses of the radio wave at the two observatories, using interferometric methods. From the phase difference, and since the radio wave is known to travel with the velocity of light, the accurate difference in distance to the object from the two observatories can be determined. Performing this kind of distance measurement during a rotation of the Earth allows a determination of the distance between the observatories themselves, even if they are situated on different continents. This method, invented in America, is known as *very long baseline interferometry* (VLBI). The first useful intercontinental results from VLBI observations were reported between the Onsala observatory on the European continent and a small group of observatories on the North American continent, by the American geodesist Thomas Herring and a number of co-workers (1986). They succeeded in determining the distance between the two observatories with an accuracy of the order of centimetres; see also the work by the Swedish space scientists Gunnar Elgered, Bernt Rönnäng and co-workers (1994).

Now, imagine a number of observatories on different continents on the Earth. The positions of the observatories are known from classical astronomical positioning and triangulation. As explained above, these coordinates are perturbed by more or less unknown deflections of the vertical due to anomalous masses inside the Earth. Hence the positions of the observatories, both absolutely and relative to one another, are unknown within several hundred metres. Analysing radio waves from the universe through VLBI, the distances between these observatories and, thereby, their relative positions suddenly can be determined within centimetres! In this way a global set of stations accurately positioned relative to one another may be created. However, the absolute position on the Earth of the set of stations as a whole remains to be fixed. This can, in principle, be accomplished by ”locating” the set as a whole in such a way that the deflections of the vertical are globally minimized.

In 1988 the first such global set of stations was established in international cooperation. It consisted of 34 stations, one of them being the Onsala space observatory. Their coordinates were published by the French astronomers and geodesists Claude Boucher and Zuheir Altamimi (1989), defining the initial International Terrestrial Reference Frame (ITRF). This marked the beginning of a new era of global positioning, in which satellites would become a powerful tool.

Man-made satellites had been circling the Earth since the late 1950s. If the motion of a satellite in its orbit is tracked from observatories on the Earth with known global ITRF coordinates, the corresponding coordinates of the satellite can be computed for any instant. If such satellites with known coordinates are observed from an arbitrary station on the Earth, the global ITRF coordinates of the arbitrary station can be computed. This is the basic principle of *satellite positioning*.

To allow a sufficiently accurate positioning, a satellite system had to be developed where each satellite emits a radio signal to be received on the Earth by a special receiver. Measuring the time it takes for the radio signal to travel from the satellite to the station, the distance between the satellite and the station can be calculated. Determining the distances in this way from a station on the Earth to at least three satellites will allow computing the position of the station on the Earth. (In practise four satellites are required, the additional satellite being needed for synchronizing clocks in the satellites and on the Earth.) See also Section 8.3 with Figure 8-1!

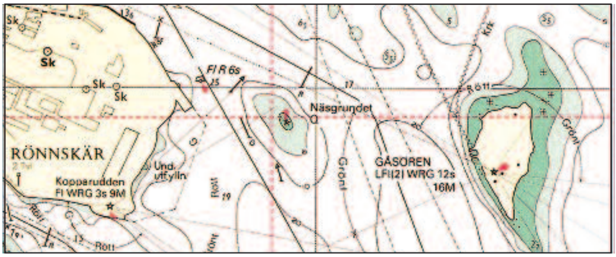

Figure 7-3. Chartlet issued in the 1990s to inform users about the shift in parallels and meridians to occur on all charts and maps when introducing the global reference system connected to satellite positioning (National Maritime Administration of Sweden).

A satellite system of the above type was developed in the 1980s by the American Defence Mapping Agency. It has become known as the Global Positioning System (GPS). Primarily it was designed for use in navigation, but is now also used in accurate mapping and for scientific investigations of the Earth. In navigation the coordinates obtained from GPS are mostly known to be in the World Geodetic System (WGS), which is a close relative of the ITRF; see Figure 7-3.

Let us summarize the *process of satellite positioning* in the following five steps:

1. Determine, from optical observations of stars on the celestial sphere (combined with triangulation), the latitudes and longitudes of a number of observatories on the Earth.

2. Determine, from radio observations of distant star systems in the Universe, the distances between the observatories of item 1.

3. Combining items 1 and 2, adjust the coordinates of the observatories into a consistent system, eliminating deflections of the vertical due to the irregular mass distribution within the Earth.

4. Determine continuously the positions of a number of satellites in their orbits by tracking them from observatories on the Earth with known coordinates according to item 3.

5. Determine the coordinates of any station on the Earth from radio observations of satellites with known positions according to item 4.

A preliminary version of satellite positioning (the Doppler method) was used at an early stage by the Nordic countries for positioning and mapping on the North Atlantic islands and in the North Sea. Early such work was performed by the Danish geodesist Frede Madsen (1978) on the Faroes and in Greenland, and by the Norwegian geodesist Jan Christian Blankenburgh (1978) in the North Sea.

To be able to use satellite positioning to its full extent most countries in the world have established and maintain a net of permanent reference stations for satellite positioning, the coordinates of which are most accurately determined in the ITRF. The Nordic countries were a pioneering area in this respect, partly because of the work performed at the Onsala space observatory. The first national net of permanent reference stations for satellite positioning was created in Sweden in 1993, its coordinates being determined by Jan Johansson and Bo-Gunnar Reit (1994). Soon after that, similar reference stations and their coordinates were established in Norway by Oddgeir Kristiansen and Bjørn Geirr Harsson (1998), in Denmark by Anna Jensen and Finn Bo Madsen (1998), and in Finland by Matti Ollikainen, Hannu Koivula and Markku Poutanen (1999). Because Sweden was so early, its coordinates have already had to be re-determined to fit better to those of the Nordic neighbours, by Lotti Jivall and Martin Lidberg (2000). The latitudes and longitudes of all these ITRF-based systems are referred to an internationally adopted geocentric ellipsoid, computed by the Austrian geodesist Helmut Moritz (1980) on the basis of satellite orbit data. (This ellipsoid actually is very close to one computed already by Helmert (1906) based on gravity data.)

Thus the end of the 1900s has seen a *revolution in positioning on the Earth, both for mapping and navigation,* comparable only to the introduction of triangulation in the middle of the 1700s. This has, however, also given rise to new problems to handle, as will be seen in the following section.

7.4 The moving pole on the Earth – and the moving continents

Back in 1844 the German astronomer Christian August Friedrich Peters, then working at the Russian central observatory of Pulkovo outside St. Petersburg, began to look for a periodic variation in its latitude. The background was that, according to Euler's theory for the rotation of rigid bodies, the axis of rotation in a freely rotating body is not stable, unless the axis of rotation coincides with an axis of symmetry of the body. Applying the theory to the Earth, being flattened at the poles, Peters (1844) found that its rotational axis should move around its symmetry axis with a period of 304 days, close to 10 months. This means that each pole of the Earth should move around in a small circle of unknown radius with this period. If so, such a *polar motion* would manifest itself as a periodic variation of the latitude with the same period.

Peters started searching for a polar motion with the predicted period of 304 days. He did not find any. His successors continued searching. They did not find any either. Still after half a century the polar motion had not been detected. Then, suddenly, a polar motion was discovered by the American insurance mathematician and private astronomer Seth Carlo Chandler (1891), reanalysing all the data. The amplitude was of the order of 10 m. But the period was not at all the predicted one – it was 427 days, close to 14 months. Already the year after Chandler's discovery the American astronomer Simon Newcomb (1892) presented an explanation of the surprisingly long period of the polar motion. From the knowledge of tides it was known that the Earth was somewhat elastic. Based on this Newcomb showed that the effect of the elasticity of the Earth is to lengthen the period of polar motion by about 100 days.

Thus the latitude of any station on the Earth, as determined by astronomical positioning, undergoes a periodical variation with an amplitude of some 10 m (0.3") and a period of 14 months. After several decades of careful observations of this phenomenon made at several stations, an additional phenomenon was discovered by the American geodesist Walter Lambert (1922). The latitudes turned out to change gradually with time, implying a secular drift of the pole in a direction towards northeastern Canada. This *polar drift* had been anticipated by Helmert (1884), suggesting, quite correctly, that it would be a consequence of redistribution of matter in the Earth in connection with the postglacial rebound of northern Canada and Scandinavia. The polar drift amounts to some 10 m per century.

As long as absolute positions, because of the deflections of the vertical, were uncertain on the order of hundreds of metres, polar motion and polar drift were not crucial problems. But with the introduction of satellite positioning, reducing absolute uncertainties to the order of centimetres, polar motion and polar drift become necessary to handle. The solution concerning the ITRF coordinates has been to fix the pole at its mean position around 1900, known as the conventional terrestrial pole. Thus all coordinates from satellite positioning so far refer to the mean pole of 1900.

However, not only the pole, together with the equator, the parallels and the meridians, move around on the Earth. Also the continents move, a phenomenon known as *continental drift* (plate tectonics). Although suggested by Wegener in the early 1900s, it was not until the introduction of VLBI that continental drift could be discovered by direct measurements. The first confirmation of that kind was a result of the Swedish-American cooperation between the Onsala observatory and the small group of American observatories. From repeated VLBI determinations of the distance between the observatories during five years, Herring et al (1986) concluded that the distance between the North American and European continents increased by 2 cm per year. We now know that the velocities of continental drift in general amount to several cm per year, corresponding to several m per century. The driving force behind this appears to be convection currents in the interior of the Earth.

Obviously, the problem of continental drift in satellite positioning has to be handled in some way. As with the polar drift the solution in the ITRF coordinates has been to fix the continents at their positions at a certain year, in this case 1989. Thus all coordinates from satellite positioning so far refer to the locations of the continents in 1989. (WGS coordinates for navigation, however, refer to more modern years.)

So, while the satellite revolution in positioning has made it possible to find one's position on the Earth with unprecedented accuracy, it has at the same time revealed new complications in specifying the position: The coordinate system itself in the form of parallels and meridians is not stable. And the continents on which we live are not stable either. Also in the vertical direction the Earth's surface is moving, because of postglacial rebound caused by the Ice Age, and because of earth tides caused by the moon and the sun – but that is another story.

7.5 A brief review

Let us now look back and make a few brief reflections on the fundamental positioning for mapping:

1. Present-day satellite positioning is in principle quite similar to old-time astronomical positioning: An institute first determines the coordinates of celestial objects and make them public. A map-maker (or navigator) then observes the same celestial objects and, using their coordinates, determines the coordinates of the observation points on the Earth.

2. On the other hand, two things have changed completely: One now uses moving objects in orbits around the Earth, satellites, instead of fixed objects on the celestial sphere, stars. And one now measures distances, in space, instead of angles, on the celestial sphere (and in a triangulation network).

3. In spite of the switch from stars to satellites, also today's satellite positioning rests on earlier observations of stars (and star systems). Thus, ultimately, it is the stars of the Universe that still make it possible to find one's position on the Earth.

In Tables 7-1 and 7-2 we have collected our estimated uncertainties of the fundamental positioning for mapping through history. Table 7-1 gives the uncertainties in the national or continental latitudes and longitudes in a relative sense. We note the drop in relative uncertainty with the introduction of triangulation added to the astronomical positioning in the middle of the 1700s (especially in longitude), and the drop again with the introduction of satellite positioning towards 2000. Table 7-2 gives the uncertainties in the global latitudes and longitudes in an absolute sense. Here we note the drastic drop in absolute uncertainty with the introduction of VLBI (using distant star systems) and satellite positioning towards 2000, eliminating the unknown deflections of the vertical. In total, the uncertainty in positioning on the Earth's surface has decreased by seven orders of magnitude during 500 years, from 10^5 m or 100 km (≈ 1°) in the 1500s to 10^{-2} m or 1 cm (≈ 0.001″) around the year 2000!

Table 7-1. Orders of magnitude of uncertainties (in m) in relative national/continental latitudes and longitudes during five centuries. (A = astronomical positioning, T = triangulation, V = VLBI, S = satellite positioning.)

Century	Lat.	Long.
1500s (A)	10^5	$> 10^5$
1600s (A)	10^4	$> 10^5$
1700s (A)	10^3	10^4
1700s (A + T)	10^2	10^2
1800s (A + T)	10^1	10^1
1900s (A + T)	10^0	10^0
2000 (A + V + S)	10^{-2}	10^{-2}

Table 7-2. Orders of magnitude of uncertainties (in m) in absolute global latitudes and longitudes during five centuries. (A = astronomical positioning, T = triangulation, V = VLBI, S = satellite positioning.)

Century	Lat.	Long.
1500s (A)	10^5	$> 10^5$
1600s (A)	10^4	$> 10^5$
1700s (A)	10^3	10^4
1800s (A)	10^3	10^3
1900s (A + T)	10^2	10^2
2000 (A + V + S)	10^{-2}	10^{-2}

8. Some special aspects

8.1 Modern coordinates of old fundamental observatories

Determining a latitude by astronomical positioning means measuring vertical angles towards a star. When putting up the instrument for measuring angles it is adjusted with a spirit level. The spirit level ”feels” the direction of the plumb line, or the vertical. The vertical, being the normal to the geoid, deviates from the normal to the ellipsoid. This deviation, the deflection of the vertical, directly affects the astronomically determined latitude, as was explained in Chapter 7.

Determining a latitude by satellite positioning means measuring distances through timekeeping of radio waves emitted from the satellites. This procedure is independent of any spirit level and, hence, does not depend on the direction of the vertical. Thus the latitude so determined is unaffected by the deflection of the vertical. The same arguments go for longitudes.

Denoting the *star-derived or astronomical latitude and longitude* by Φ and Λ, and the *satellite-derived or geodetic latitude and longitude* by φ and λ, we may write, as in Section 7.1,

$$\Phi = \varphi + \xi \tag{8-1}$$
$$\Lambda = \lambda + \eta / \cos \varphi \tag{8-2}$$

Here ξ and η are the *absolute deflections of the vertical* in the south-north and west-east directions, respectively.

Now, the deflection of the vertical at a certain point is nothing but the inclination of the geoid relative to the ellipsoid at that point. The geoid height and, thereby, the deflections of the vertical are due to the irregular mass distribution within the Earth. Hence these quantities can be computed from a detailed and global knowledge of the Earth’s gravity field. Such a knowledge has only been achieved during the last decades. Modern geoid computations are based on a combination of satellite orbit perturbations, surface gravity anomalies, and digital terrain models.

Table 8-1. Geodetic coordinates, deflections of the vertical, and astronomical coordinates (in degrees, minutes and seconds) of some fundamental observatories. Longitude deflections divided by cos φ.

Observatory	Geod. lat.	Defl.	Astr. lat.
Stockholm	59° 20′ 29.16″	3.89″	59° 20′ 33.0″
København	55 40 53.06	0.27	55 40 53.3
Greenwich	51 28 40.12	- 2.15	51 28 38.0

Observatory	Geod. long.	Defl.	Astr. long.
Stockholm	18° 03′ 16.76″	12.69″	18° 03′ 29.4″
København	12 34 32.79	5.94	12 34 38.7
Greenwich	- 0 00 05.31	5.51	0 00 00.2

Applying the above methods the author and his Swedish colleague Jonas Ågren have used satellite positioning and gravimetric deflections of the vertical to calculate the astronomical latitudes and longitudes of some of the old fundamental observatories. The geodetic coordinates of the observatories are determined from satellite positioning performed on neighbouring triangulation stations combined with local ties to the observatories. The deflections of the vertical are computed as the derivatives of the geoid height in the south-north and west-east directions, respectively. In a first paper (Ekman & Ågren, 2009) the latitude of the Uranienborg observatory was studied. In a second paper (Ekman & Ågren, 2010) the latitudes and longitudes of the København and Stockholm observatories were investigated, and also those of the Greenwich observatory. The results are summarized in Table 8-1. The geodetic coordinates there are in an ITRF-related system and can be considered error-free. The deflections of the vertical have an estimated uncertainty of 0.2″; accordingly the astronomical coordinates will have the same uncertainty. Thereby the quality of the old astronomical observatory coordinates may be investigated through an independent method.

The results in Table 8-1 are used to produce the modern coordinates for Tables 5-2 and 5-4 in Chapter 5 and for Table 6-1 in Chapter 6. (In Tables 5-4 and 6-1 the modern longitudes for the other observatories have been estimated by adding the telegraph-determined longitude differences from København and Stockholm, respectively.) The tables show that the errors in observatory latitude fell below 1″ with the determinations of Schumacher (1827a) and Selander (1835), and that the errors in observatory longitude fell below the same amount with the sea expeditions of Struve (1844, 1846).

8.2 A triangulation lost and retrieved

After Finland and Åland had been ceded by Sweden to Russia in 1809, the Russians decided to build a large fortress on Åland and to move a part of their Baltic naval fleet to these islands. This might be a background for a peculiar Russian interest in the coordinates from the triangulation across Åland treated in Chapter 5. What happened was the following.

The original report of Gadolin (1757) was for unknown reasons never printed. When Hällström (1815) recomputed Gadolin's triangulation, his report was for unknown reasons not printed either. After Hällström had died, his brother sent his manuscript of the report to the Finnish Society of Sciences, in 1839. Soon after that, the Russian navy hastened to borrow the manuscript but never returned it. After some years they returned a copy of it instead, certified by two persons to be reasonably true. Since the original manuscript now had become inaccessible it was not printed this time either. Not until at the end of the century was Hällström's (1815) report printed, but now based on the Russian copy of it. The original manuscript probably no longer exists.

However, an earlier version of the original manuscript does in fact exist, in the Geodetic Archives of the National Land Survey of Sweden. This can be seen to be the ″pre-original″ manuscript, since Hällström here in a few cases has left an empty space in the text for later filling in missing details. Moreover, a complete manuscript is kept in the Archives of the National Maritime Administration of Sweden. The present author, therefore, now has compared the coordinates in the pre-original as well as in the complete manuscript with those in the Russian copy. After nearly 200 years it can be confirmed that almost all coordinates of the Russian copy are correct. (In one case, Getaberg, the longitude is in error by 5′, but this error occurs already in the complete manuscript.)

The Russian interest for this triangulation may be considered as a further illustration of its accuracy: When the Russian navy got hold of the document and did not want to return it, the measurements in the document were nearly 100 years old, but obviously still considered valuable for their hydrographers.

8.3 Ships instead of satellites

The basic principles of satellite positioning, introduced through the Global Positioning System (GPS), have been described in Chapter 7. They could be listed, in simplified form, as follows:

1. No sight between geodetic stations is needed.
2. From each station a number of distant moving objects – satellites – are observed.
3. For each moving object the distance between the object and the station is determined.
4. The distance is found by measuring the travelling time of a wave with a known speed – a radio wave – emitted from the moving object.

Although the above method for positioning may seem modern, its basic principles can, in fact, be said to have been proposed more than 250 years ago. In a short paper, briefly mentioned in Chapter 5, Meldercreutz (1741) proposed a positioning method based on principles that may be formulated in the following way:

1. No sight between geodetic stations is needed.
2. From each station a number of distant moving objects – war-ships – are observed.
3. For each moving object the distance between the object and the station is determined.
4. The distance is found by measuring the travelling time of a wave with a known speed – a sound wave – emitted from the moving object.

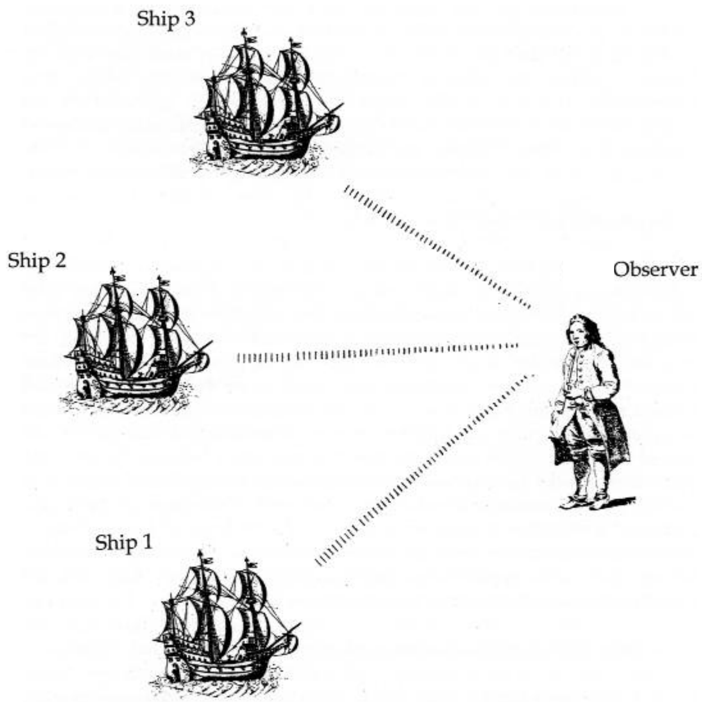

Figure 8-1. Meldercreutz´ idea of coastal positioning. If we exchange the ships for satellites we get the idea behind satellite positioning.

As can be seen, these principles can be said to be the same as for GPS. Only the technology is somewhat older: War-ships instead of satellites are suggested, and sound waves instead of radio waves; see Figure 8-1. Let us call this method CPS, Coastal Positioning System.

Around the Baltic Sea there was at that time a great need for better coastal maps and nautical charts. Triangulation had not yet been applied for this purpose. Especially the important travelling and postal route between Sweden and Finland through the Åland Islands with its extensive archipelago was in need of a more reliable mapping. Meldercreutz had married a girl from the Finnish side of the Baltic, which made him travel several times between Sweden and Finland across the Åland Islands, probably along the mentioned route (it is known that he died on Åland along this route). Here he must have had unlimited possibilities to experience the lack of knowledge of the positions of islands and coasts;

there was no reasonably accurate map or chart available. When, later on, triangulation was introduced for mapping the Baltic Sea, the Åland Islands was the first area to be measured, as shown in Chapter 5.

According to Meldercreutz (1741), there could be different ways to apply CPS. One way would be to determine the position of a coastal station through the observations of at least two war-ships (and also the position of a ship through corresponding observations of at least two coastal stations). Another way would be to determine the relative positions of two coastal stations through simultaneous observations of the war-ships from both stations. Again we recognize similarities with GPS, now in the form of absolute and relative GPS. Meldercreutz suggested the possibility of using a whole fleet of war-ships for determining the (relative) positions of a whole set of stations along the coast.

In order to determine the distances from the war-ships to a station one was supposed to use a sound wave through the air. The sound would be produced by firing a cannon on board the ship. This facilitates time-keeping: When you fire the cannon, the observer at the station on the coast will immediately see the light of the flame and start recording the time. A number of seconds later the thunder from the cannon will be heard by the observer, who will then measure the time elapsed, i.e. the travelling time of the sound wave. Thus, time-keeping in CPS is sufficient to do at the station, while in GPS time-keeping is needed both at the station and in the satellites (requiring an extra satellite for calibration of clocks).

To calculate the distance from the measured travelling time one would then need to know the speed of the sound wave through the air. Meldercreutz recommended for the speed of sound a value based on British, French and Italian determinations, which was only 1 – 2 % larger than the modern value.

However, there is no sign of CPS ever having been applied in real field work. Why? One can assume that there must have been two problems.

The first problem would be to determine the positions of the war-ships. This is a problem also with the satellites in GPS. In contrast to a satellite, however, it is hardly possible to compute the motion of a sailing war-ship in advance with sufficient accuracy, i.e. there is no reliable ”broadcast ephemeris”. On the other hand, there could be a kind of ”precise ephemeris”, where you afterwards calculate the ship’s position from actual measurements on board the ship. If the depth of the water is not too large, this could be made easier by casting anchor while the measurements are going on, a possibility not available for satellites. Still, great efforts would be needed to obtain sufficiently accurate ship positions.

The second problem would be to measure the time with sufficient accuracy. This was no doubt the main problem. Supposing a ship to be located, say, 5 nautical miles (nearly 10 km) off the coast, the sound of a fired cannon would take some 30 seconds to reach the coast. To be useful this would require the time to be measured with an accuracy of fractions of seconds. This was not possible in the field with the clocks available then.

In summary, the idea presented at this early stage in the development of positioning is quite modern, although it could not be realized with sailing war-ships and fired cannons, but had to await satellites and radio signals!

8.4 Making maps with needles

In Chapter 6 we gave a brief explanation of the way triangulations were used to put objects in their correct places on a national map. In principle the fundamental triangulation was densified by local triangulations, which in their turn were densified by even more local measurements of directions and distances. In this way everything could be placed on the map with their correct coordinates and the whole map or chart constructed.

In Sweden, however, the situation was somewhat special. Sweden happened to have a unique collection of old local maps of farms and villages. These maps did not have any common coordinate system and could thus not be accurately connected or related to each other. On the other hand it seemed a waste not to utilize all the geographic information in these maps when producing the national topographic map in the 1800s. So, what was done was the following.

The parallels and meridians of a topographic map sheet were constructed on a large sheet of paper fastened on a large table. Relevant local maps were copied and diminished to small map fragments having a suitable scale for being put together as a basis for the map sheet. Now, all available triangulation stations were marked in their proper locations on these map fragments or small pieces of paper. Each piece of paper containing a triangulation station was fixed onto the table with a needle through the triangulation station, the needle being put down into the table at the correct latitude and longitude of the map sheet. Pieces without a triangulation station had to be adjusted to their triangulated neighbours through overlapping. The scene is described by a contemporary geodesist: ”A table 3 or 4 ells [about 2 m] square is found heaped with more than one thousand scraps of paper, of a size from that of a thumbnail to that of a man’s hand, all of which so far are kept in place by a forest of varnished sewing needles.” Then these map fragments were glued together to form a preliminary map sheet for further mapping work.

Although unusual, this method nicely illustrates the fundamental role played by the astronomical and geodetic work in making a map: The triangulation stations – or nowadays the satellite stations – constitute the skeleton, to which everything else in the map is tied!

References (in chronological order)

Helgason, O (c. 1150): Stjörnu-Odda tal. Manuscript, Flatey, Iceland. (Royal Library, København; also partly reprinted in German translation in O S Reuter: Germanische Himmelskunde, J F Lehmanns Verlag, 1934.)

Mercator, G (1569): Nova et aucta orbis terrae descriptio ad usum navigantium emendate accommodata. Map, Duisburg.

Ortelius, A (1570): Septentrionalium regionum descriptio. Map with text in: Theatrum orbis terrarium, Antwerpen.

Brahe, T (1596): Epistolarum astronomicarum libri. Uranienborg, 309 pp.

Brahe, T (1598): Astronomiae instauratae mechanica. Wandsbeck, c. 150 pp (no page numbers).

Brahe, T (1602): Astronomiae instauratae progymnasmata. Prag, 822 pp.

Bure, A (1626): Orbis arctoi nova et accurata delineatio. Map with text, Stockholm. (Republished 1635 in a smaller version under a different title in W Blaeu's Novus atlas, later Atlas maior, Amsterdam.)

Kepler, J (1627): Tabulae Rudolphinae, quibus astronomicae scientiae temporum longinquitate collapsae restauratio continetur. Ulm, 239 pp.

Blaeu, W, & Blaeu, J (1638): Insula Hvaena sive Venusia. Map in: Novus atlas, later Atlas maior, Amsterdam.

Hedraeus, B (1643): Nova et accuratae astrolabii geometrici structura nec non quadrantis astronomici azimuthalis, una cum utriusqve usu. Leiden, 104 pp.

Bartholin, R (1672): Elevatio poli seu latitudo geographica Hauniensis, & Longitudo geographica vera urbis Hauniensis. Acta medica et philosophica Hafniensia, 1671 & 1672, 217-219.

Rømer, O (1676): Demonstration touchant le movement de la lumiere. Journal des Scavans, 1676, 267-270.

Picard, J (1680): Voyage d'Uranibourg, ou observations astronomiques faites en Dannemarck. Paris, 29 pp.

Newton, I (1687): Philosophiae naturalis principia mathematica. London, 510 pp.

von Rosenfeldt, W, & Gedda, P (1694): Wäxande grad-charta öfwer Östersiön ifrån Skagen till Norrebottn och Nyenskantz. Chart, Amsterdam.

Bilberg, J (1695): Refractio solis inoccidui, in septemtrionalibus oris / Midnats solens rätta och synlige rum uti Norrlanden. Stockholm, 103 pp.

Flamsteed, J (1725): Historia coelestis Britannica, 3 vol. London.

Horrebow, P (1735): Basis astronomiae sive astronomiae pars mechanica, sive methodi observandi Roemerianae. København, 232 pp.

de Maupertuis, P L M, Clairaut, A C, Camus, C E L, Le Monnier, P C, Outhier, R, Celsius, A (1738): La figure de la Terre, determinée par les observations faites par ordre du Rois au Cercle polaire. Paris, 216 pp. (Also in English: The figure of the Earth, determined from observations made by order of the French king at the Polar circle. London, 232 pp.)

Celsius, A (1739): Elevationem veram poli octante circuli non minorem, sine cognitione refractionis siderum, invenire. Acta literaria et scientiarum Sveciae, 1739, 453-454. (Accompanying observations published by the French Academy of Sciences, but for some reason not until 1763.)

Cassini, J (1740): Tables astronomiques du soleil, de la lune, des planetes, des étoiles fixes, et des satellites de Jupiter et de Saturne. Paris, 342 pp.

Celsius, A (1741): Longituden af Köpenhamn wester om Upsala meridian determinerad. Kongl. Swenska Wetenskaps Academiens Handlingar, 2, 167-168.

Celsius, A, & Hiorter, O (1741): Anmärkning om nyttan at weta Jordens rätta storlek och figur. Kongl. Swenska Wetenskaps Academiens Handlingar, 2, 259-265.

Meldercreutz, J (1741): Om längders mätning genom dåns tilhielp. Kongl. Swenska Wetenskaps Academiens Handlingar, 2, 73-77.

Wargentin, P (1741): Tabulae pro calculandis eclipsibus satellitum Jovis, ad meridianum Observatorii Upsaliensis. Acta societatis regiae scientiarum Upsaliensis, 1741, 27-66. (Improved complete versions published in 1759 in J J Lalande's Tables astronomiques pour les planets et les comets, and in 1776 in J E Bode's Sammlung astronomischer Tafeln.)

Celsius, A (1743): Longituden af Torneå öster om Upsala meridian determinerad. Kongl. Swenska Wetenskaps Academiens Handlingar, 4, 153-157.

Clairaut, A C (1743): Theorie de la figure de la Terre, tirée des principes de l'hydrostatique. Paris, 305 pp.

Cassini de Thury, C F (1744): La meridienne de l'observatoire royal de Paris, vérifiée dans toute l'étendue du royaume par de nouvelles observations; pour en déduire la vraye grandeur des degrés de la Terre & pour lever une carte générale de la France. Paris, 399 pp. (Map of triangulation network with coordinates separately published.)

Biurman, G (1747): Svea ock Göta riken med Finland ock Norland afritade. Map, Stockholm.

Bradley, J (1748): A letter concerning an apparent motion observed in some of the fixed stars. Philosophical Transactions of the Royal Society of London, 45, 1-43.

Hellant, A (1750): Astronomiska observationer på åtskilliga orters polhögder innom pol-cirkelen. Kongl. Svenska Vetenskaps Academiens Handlingar, 11, 218-226.

Hellant, A (1752): Geographiska longituden af någre orter inom polcirkelen, bestämd genom astronomiska observationer. Kongl. Svenska Vetenskaps Academiens Handlingar, 13, 228-238.

Gadolin, J (1753): Åbo stads belägenhet bestämd genom observationer. Kongl. Svenska Vetenskaps Academiens Handlingar, 14, 266-272.

Schenmark, N (1754): Hernösands stads belägenhet bestämd genom observationer. Kongl. Svenska Vetenskaps Academiens Handlingar, 15, 70-77.

Mayer, A (1756): Astronomiska observationer angående Greifswalds stads geographiska belägenhet. Kongl. Svenska Vetenskaps Academiens Handlingar, 17, 224-230.

Wargentin, P (1756): Försök at determinera solens parallaxis, genom correspondenta observationer på planeten Mars, hållne år 1751 vid Caput Bonæ Spei af herr de la Caille och i Stockholm af Pehr Wargentin. Kongl. Svenska Vetenskaps Academiens Handlingar, 17, 60-72.

Gadolin, J (1757): Triangelmätning ifrån Åbo till Grisselhamn. Official report to the Royal Survey Office, Stockholm. (Archives of the National Maritime Administration of Sweden, Norrköping. Recomputed by Hällström (1815) below.)

Wargentin, P (1759): Stockholms observatorii polhögd bestämd. Kongl. Vetenskaps Academiens Handlingar, 20, 305-314.

Wargentin, P (1761): Meridian-skillnaden imellan observatorierne i Paris och Stockholm, bestämd genom observationer. Kongl. Vetenskaps Academiens Handlingar, 22, 247-251.

Schenmark, N (1765): Några orters geographiska belägenhet omkring Uraniburg. Kongl. Vetenskaps Academiens Handlingar, 26, 58-69.

Wessel, C (1768): Den nord-østlige Fierdedeel af Siæland ved rigtig Landmaaling optaget og ved trigonometriske samt astronomiske Operationer prøvet. Map by the Royal Academy of Sciences, København.

Lambert, J H (1772): Anmerkungen und Zusätze zur Entwerfung der Land- und Himmelscharten. In: Beyträge zum Gebrauche der Mathematik und deren Anwendung, 3. Berlin.

Lexell, A J (1773): Några svenska orters geographiska longituder, uträknade af observationer öfver sol-förmörkelserne åren 1764 och 1769. Kongl. Vetenskaps Academiens Handlingar, 34, 44-66 & 117-134 (with corrections in 35, 170-172).

Wargentin, P (1773): Om Uraniburgs och Lunds observatorii longitud wester om Stockholms meridian. Kongl. Vetenskaps Academiens Handlingar, 34, 66-71.

Schenmark, N (1774 & 1780): De märkvärdigaste orters geographiska belägenhet, vid sjökusten i Skåne, Halland och Bohus-län. Kongl. Vetenskaps Academiens Handlingar, 35 & 41, 205-230 & 74-81.

Wargentin, P (1777): A letter concerning the difference of longitude of the royal observatories at Paris and Greenwich, resulting from the eclipses of Jupiter's first satellite observed during the last ten years. Philosophical Transactions of the Royal Society of London, 67, 162-186. (In Latin although title in English.)

Bugge, T (1779): Beskrivelse over den Opmaalings Maade som er brugt ved de danske geographiske Karter, med tilføiet trigonometrisk Karte over Siæland, og de der henhørende Triangler, beregnede Longituder og Latituder, samt astronomiske Observationer. København, 132 pp.

Nordenankar, J (1783): Special charta öfver kusten från Söderhamn til Stockholm författad efter astron. observationer, trigon. och hydrogr. mätningar. Chart by the Royal Admiralty, Stockholm.

Bugge, T (1784): Observationes astronomicae annis 1781, 1782 & 1783. København, 261 pp.

Pontoppidan, C I (1785): Kart over det sydlige Norge. Map, København.

Maskelyne, N (1787): Concerning the latitude and longitude of the Royal Observatory at Greenwich; with remarks on a memorial of the late M. Cassini de Thury. Philosophical Transactions of the Royal Society of London, 77, 151-187.

af Wetterstedt, E (1789): Charta öfver Åland med en del af svenska och finska skären samt post-vägen imellan Sverige och Finland efter geometr. och trigon. mätningar samt astron. observationer författad. Map by the Royal Survey Office, Stockholm.

Wessel, C (1797): Om Directionens analytiske Betegning, et Forsøg, anvendt fornemmelig til plane og sphaeriske Polygoners Opløsning. Kongel. Danske Videnskabernes Selskabs Skrifter, 5, 469-518.

Schultén, N G (1801): Försök at genom astronomiska och chronometriska observationer närmare bestämma Östersjöns bredd, Gottlands rätta läge och flere orters geographiska belägenhet. Kongl. Vetenskaps Academiens Handlingar, 22, 271-298.

Svanberg, J (1805): Exposition des opérations faites en Lapponie pour la détermination d'un arc du méridien en 1801, 1802 et 1803. Stockholm, 196 pp.

Cronstrand, S A (1811): Stockholms observatorii polhögd beräknad. Kongl. Vetenskaps Academiens Handlingar, 32, 291-295.

Hällström, C P (1815): Triangelmätning ifrån Åbo, öfver Åland, till Stacksten på svenska kusten, förrättad af Jac. Gadolin, ånyo uträknad. Official report, Stockholm. (Geodetic Archives at the National Land Survey of Sweden, Gävle, and Archives of the National Maritime Administration of Sweden, Norrköping; also reprinted in a slightly different version in Acta Societatis Scientiarum Fennicae, 20/2, 1893.)

Spens, C G (1817): Försök att bestämma den tjenligaste projections-methoden för land-chartor öfver mindre delar af jordytan; jemte beskrifning på ett efter denna method uppgjordt projectionsnät öfver Skandinavien. Kongl. Vetenskaps Academiens Handlingar, 1817, 161-197.

Walbeck, H J (1819): De forma et magnitudine Telluris, ex dimensis arcubus meridiani, definiendis. Dissertation, Åbo [Turku], 16 pp.

Gauss, C F (1827): Chronometrische Längenbestimmungen. Astronomische Nachrichten, 5, 227-234.

Schumacher, H C (1827): Längenunterschiede zwischen Altona, Bremen, Helgoland und Greenwich, durch Chronometer bestimmt. Astronomische Nachrichten, 5, 225-240 & 245-248.

Schumacher, H C (1827a): De latitudine speculae Havniensis. Altona, 20 pp.

Gauss, C F (1828): Bestimmung des Breitenunterschiedes zwischen den Sternwarten von Göttingen und Altona durch Beobachtungen am Ramsdenschen Zenithsector. Göttingen, 84 pp.

Hansen, P A (1831): Längenbestimmungen zwischen Altona, Hamburg, Lübeck und Copenhagen durch Chronometer. Astronomische Nachrichten, 8, 269-282.

Schumacher, H C (1831): Längendifferenz zwischen Copenhagen und Altona. Astronomische Nachrichten, 9, 163-164.

Selander, N H (1835): Undersökning om Stockholms observatorii polhöjd. Kongl. Vetenskaps Academiens Handlingar, 1835, 170-204.

Schubert, T F (1836): Khronometricheskaya expditsiya proizvedennaya po vysochayshemu yego imperatorskago velichestva poveleniyu v 1833 godu, 2 vol. St. Petersburg, 258 & 180 pp.

Argelander, F (1837): Auszug aus einem Schreiben des Herrn Professors Argelander, Directors der Sternwarte in Helsingfors, an den Herausgeber. Astronomische Nachrichten, 14, 203-206.

Bessel, F W (1837 & 1841): Bestimmung der Axen des elliptischen Rotations-sphäroids, welches den vorhandenen Messungen von Meridianbögen der Erde am meisten entspricht. Astronomische Nachrichten, 14 & 19, 333-339 & 97-104.

Peters, C A F (1844): Resultate aus Beobachtungen des Polarsterns am Ertelschen Verticalkreise der Pulkowaer Sternwarte. Astronomische Nachrichten, 22, 119-128.

Struve, F G W (1844): Expédition chronométrique excécutée par ordre de sa majesté l'empereur Nicolas Ier entre Poulkova et Altona pour la détermination de la longitude géographique relative de l'observatoire central de Russie. St. Petersbourg, 152 pp.

Struve, F G W, & Struve, O W (1846): Expédition chronométrique excécutée par ordre de sa majesté l'empereur Nicolas Ier entre Altona et Greenwich pour la détermination de la longitude géographique de l'observatoire central de Russie. St. Petersbourg, 206 pp.

Hansteen, C, & Fearnley, C (1849): Beschreibung und Lage der Universitäts-Sternwarte in Christiania. Christiania [Oslo], 88 pp.

Stokes, G G (1849): On the variation of gravity at the surface of the Earth. Transactions of the Cambridge Philosophical Society, 8, 672-695.

Struve, F G W (1857 & 1860): Arc du méridien de 25°20′ entre le Danube et la Mer Glaciale, 1 & 2. St. Petersbourg, 334 & 483 pp.

Selander, N H (1866): Ortbestämmelser i Sverige verkställde af Topgrafiska Corpsen åren 1814–1849. Stockholm, 54 pp. (Author anonymous.)

Andræ, C G (1867): Hovedtrianglerne paa Sjælland og deres Forbindelser med svenske og preussiske Triangelrækker. Den Danske Gradmaaling, 1, 579 pp.

Fuss, V, & Nyrén, M (1871): Bestimmung der Längen-Differenz zwischen den Sternwarten Stockholm und Helsingfors. Mémoires de l'Académie Impériale des Sciences de St.-Pétersbourg, 17/10, 36 pp.

Andræ, C G (1872): Meridianbuens Hovedtriangler fra Elben til Samsø og deres Forbindelse med Maalingerne paa Sjælland. Den Danske Gradmaaling, 2, 490 pp.

Andræ, C G (1878): De tilbagestaaende Dele af Triangelnettet og dettes Nedlægning paa Sphæroiden. Den Danske Gradmaaling, 3, 422 pp.

Rosén, P G (1879): Om de geodetiska och astronomiska ortbestämmelserna i Sverige. Svenska sällskapet för antropologi och geografi, Geografiska sektionens tidskrift, 1/9, 28 pp. (With summary in French: Sur les déterminations de lieu géodésiques et astronomiques exécutées en Suède.)

Helmert, F R (1880 & 1884): Die mathematischen und physikalischen Theorieen der höheren Geodäsie, 2 vol. Leipzig, 631 & 610 pp.

Andrae, C G, & Peters, C A F (1884): De astronomiske Iakttagelser og Bestemmelsen af Sphæroiden. Den Danske Gradmaaling, 4, 432 pp.

Peters, C A F (1884): Bestimmung des Längenunterschiedes zwischen den Sternwarten von Kopenhagen und Altona. In Andrae, C G, & Peters, C A F: De astronomiske Iakttagelser og Bestemmelsen af Sphæroiden. Den Danske Gradmaaling, 4, 1-146.

Helmert, F R (1886): Lothabweichungen – Heft 1: Formeln und Tafeln sowie einige numerische Ergebnisse für Norddeutschland. Veröffentlichung des Königl. Preussischen Geodätischen Instituts, 120 pp.

Rosén, P G (1889): Om lodafvikelser i Sverige. Bihang till Kongl. Svenska Vetenskaps-Akademiens Handlingar, 15/1:7, 10 pp.

Lindhagen, D G, Fearnley, C F, & Schjellerup, F C (1890): Längenbestimmungen zwischen den Sternwarten in Stockholm, Kopenhagen und Christiania. Kongl. Svenska Vetenskaps-Akademiens Handlingar, 24/4, 52 pp.

Chandler, S C (1891): On the variation of latitude. Astronomical Journal, 249, 65-70.

Newcomb, S (1892): On the dynamics of the Earth's rotation, with respect to the periodic variations of latitude. Monthly Notices of the Royal Astronomical Society, 52, 336-341.

Geelmuyden, H (1895): Astronomische Beobachtungen und Vergleichung der astronomischen und geodätischen Resultate. Publication der Norwegischen Commission der Europäischen Gradmessung, 97 pp.

Börsch, A (1906): Lotabweichungen – Heft III: Astronomisch-geodätisches Netz I. Ordnung nördlich der Europäischen Längengradmessung in 52 Grad Breite. Veröffentlichung des Königl. Preuszischen Geodätischen Institutes, 28, 164 pp.

Helmert, F R (1906): Die Grösse der Erde. Sitzungsberichte der Königl. Preussischen Akademie der Wissenschaften, 1906, 525-537.

Hayford, J F (1909 & 1910): The figure of the Earth and isostasy from measurements in the United States. Coast and Geodetic Survey, 178 & 80 pp.

Krüger, L (1912): Konforme Abbildung des Erdellipsoids in der Ebene. Veröffentlichung des Königl. Preuszischen Geodätischen Institutes, 52, 172 pp.

Jäderin, E (1915): Mesure de la base. Missions scientifiques pour la mesure d'un arc de méridien au Spitzberg enterprises en 1899–1902, 2A, 229 pp.

Lambert, W D (1922): The interpretation of apparent changes in mean latitude, at the international latitude stations. Astronomical Journal, 804, 103-110.

Ölander, V R (1931): Über die Beziehung zwischen Lotabweichungen und Schwereanomalien sowie über das Lotabweichungssystem in Süd-Finnland. Veröffentlichungen des Finnischen Geodätischen In stitutes, 17, 23 pp.

Bonsdorff, I (1944): Die astronomisch-geodätischen Arbeiten während der Sonnenfinsternis den 9. Juli 1945. Die Tätigkeit der Baltischen Geodätischen Kommission in den Jahren 1942-43, 12-16.

Väisälä, Y (1946): An astronomical method of triangulation. Sitzungsberichte der Finnischen Akademie der Wissenschaften, 1946, 99-107.

Bergstrand, E (1948): Measurement of distances by high frequency light signalling. Die Tätigkeit der Baltischen Geodätischen Kommission in den Jahren 1944-47, 101-111.

Ölander, V R (1949): Adjustment of the Baltic ring. Special Publication of the Baltic Geodetic Commission, 10, 68 pp.

Simonsen, O (1949): Preliminary report on the triangulation between Denmark and Norway carried out in 1945 by means of parachute flares. Bulletin Géodésique, 11, 33-52.

Wolf, H (1949): Beiträge zur Lotabweichungsausgleichung und Geoidbestimmung – Allgemeine Betrachtungen und Durchführung im Bereich des Zentraleuropäischen Netzes. Veröffentlichungen des Instituts für Erdmessung, 6, 106 pp.

Bergstrand, E (1950): A determination of the velocity of light. Kungl. Svenska Vetenskapsakademiens Arkiv för Fysik, 2, 119-150.

Whitten, C A (1952): Adjustment of European triangulation. Bulletin Géodésique, 24, 187-206.

Krarup, T, & Svejgaard, B (1956): A method for matrix multiplication, matrix inversion, and problems of adjustment by punched card equipment. Geodætisk Instituts Meddelelse, 31, 30 pp.

Väisälä, Y, & Oterma, L (1960): Anwendung der astronomischen Triangulationsmethode. Veröffentlichungen des Finnischen Geodätischen Institutes, 53, 18 pp.

Blankenburgh, J C (1978): Some achievements and research-projects based on the use of satellites. Nordiska Kommissionen för Geodesi, 8:e mötet, 197-207.

Madsen, F (1978): Anvendelse af Dopplersatellitmetoder til positionsbestemmelse. Nordiska Kommissionen för Geodesi, 8:e mötet, 126-134.

Moritz, H (1980): Geodetic reference system 1980. Bulletin Géodésique, 54, 395-405.

Herring, T A, and 17 co-authors (1986): Geodesy by radio interferometry: Evidence for contemporary plate motion. Journal of Geophysical Research, 91 B, 8341-8347.

Boucher, C, & Altamimi, Z (1989): The initial IERS terrestrial reference frame. IERS [International Earth Rotation Service] Technical Note, 1, 22 pp.

Elgered, G, Carlsson, T R, Carlsson, T M, Jaldehag, R T K, Jarlemark, P O J, Johansson, J M, Nilsson, B I, Rönnäng, B O, Scherneck, H-G (1994): The permanent geodetic station at the Onsala space observatory. Nordic Geodetic Commission, 12th General Meeting, 353-363.

Reit, B-G (1994): SWEREF 93 – A Swedish reference system for GPS. Nordic Geodetic Commission, 12th General Meeting, 401-409. (The first part of this work, by J Johansson, was never published.)

Reit, B-G (1995): Swedish coordinate systems. In Vermeer (ed): Coordinate systems, GPS, and the geoid, Reports of the Finnish Geodetic Institute, 95/4, 145-147. See also Reit, B-G (2010): On geodetic transformations. Lantmäteriverkets rapporter i geodesi och geografiska informationssystem, 2010:1, 52 pp.

Kristiansen, O, & Harsson, B G (1998): The new Norwegian national geodetic network – EUREF 89. Nordic Geodetic Commission, 13th General Meeting, 162-195.

Jensen, A B O, & Madsen, F B (1998): A new three dimensional reference network in Denmark. Nordic Geodetic Commission, 13th General Meeting, 217-224. See also Fankhauser, S, & Gurtner, W (1995): Denmark EUREF densification campaign EUREF-DK94. International

Association of Geodesy, Subcommission for the European Reference Frame, 4, p 36.

Ollikainen, M, Koivula, H, & Poutanen, M (1999): The densification of the EUREF network in Finland. International Association of Geodesy, Subcommission for the European Reference Frame, 8, 114-122.

Jivall, L, & Lidberg, M (2000): SWEREF 99 – An updated EUREF realisation for Sweden. International Association of Geodesy, Subcommission for the European Reference Frame, 9, 167-175.

Ekman, M, & Ågren, J (2009): A study of Tycho Brahe's astronomical latitude determination of Uranienborg using satellite positioning and deflections of the vertical. Small Publications in Historical Geophysics, 18, 13 pp.

Ekman, M, & Ågren J (2010): Reanalysing astronomical coordinates of old fundamental observatories using satellite positioning and deflections of the vertical. Small Publications in Historical Geophysics, 21, 18 pp.

Note: As explained in the preface the above reference list is not ordered alphabetically but chronologically.

Index

A

B

C

D

E

F

G

H

I

J

K

L

M

N

O

P

Q

R

T

U

V

W

Z

This is a book about 500 years of Nordic answers to a very specific question: Where on Earth are we? The theme of the book may be described as: How to use the sky – sun, stars, moons, satellites etc. – to find positions on the Earth in order to construct a map. What we study here is the fundamental positioning for the mapping of the Nordic countries from the 1500s up till today. This is of a wider international interest than might be assumed.

Moreover, the book has a combined scientific and historical perspective throughout. On one hand, science is used to contribute to understanding the historical development of positioning and mapping. On the other hand, the historical development is used to contribute to understanding the principles behind modern scientific positioning. Original scientific sources (and maps) are used throughout. This means throwing light also on important works no longer known to modern scientists.

Finally, in order to broaden the outlook in somewhat unexpected directions, some special aspects related to the positioning and mapping problems are included at the end of the book.

This book spans from astronomy via geodesy to mapping (and partly navigation). It is intended for reading by a wide range of geoscientists or other people with a professional interest in the mapping of the Earth. I hope you will find the book interesting.

978-952-92-91373