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A Study of Tycho Brahe's Astronomical Latitude Determination of Uranienborg using Satellite Positioning and Deflections of the Vertical

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Åland Islands

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1. The observatory of Uranienborg

In 1576 Tycho Brahe moved to the island of Ven (at that time spelt Hven), situated in the entrance to the Baltic Sea. The King of Denmark had given him the right to use this island for erecting an astronomical observatory. The building that Brahe erected, Uranienborg, was a remarkable combination of astronomical observatory and decorated palace, surrounded by a geometrical garden; see Figure 1. Later a group of small separate observatory buildings were added.

Brahe constructed his own instruments, designing them to give an accuracy never achieved before. Moreover, he checked his various instruments against each other. His main instrument was a large quadrant for measuring vertical angles in the meridian, mounted on a stable and specially painted wall in the main building; see Figure 2. With this instrument and others he could accurately measure the altitude (height) of stars above the horizon; Brahe (1598) claimed that the altitudes could be read within 1/6 of a minute (10″). Observations, involving also a number of research assistants, were performed almost every clear night during a period of 20 years – just with the naked eye, as the telescope had not yet been invented.

A fundamental task at the observatory was to determine its latitude from the altitudes of the stars. Once the latitude was known the declinations of

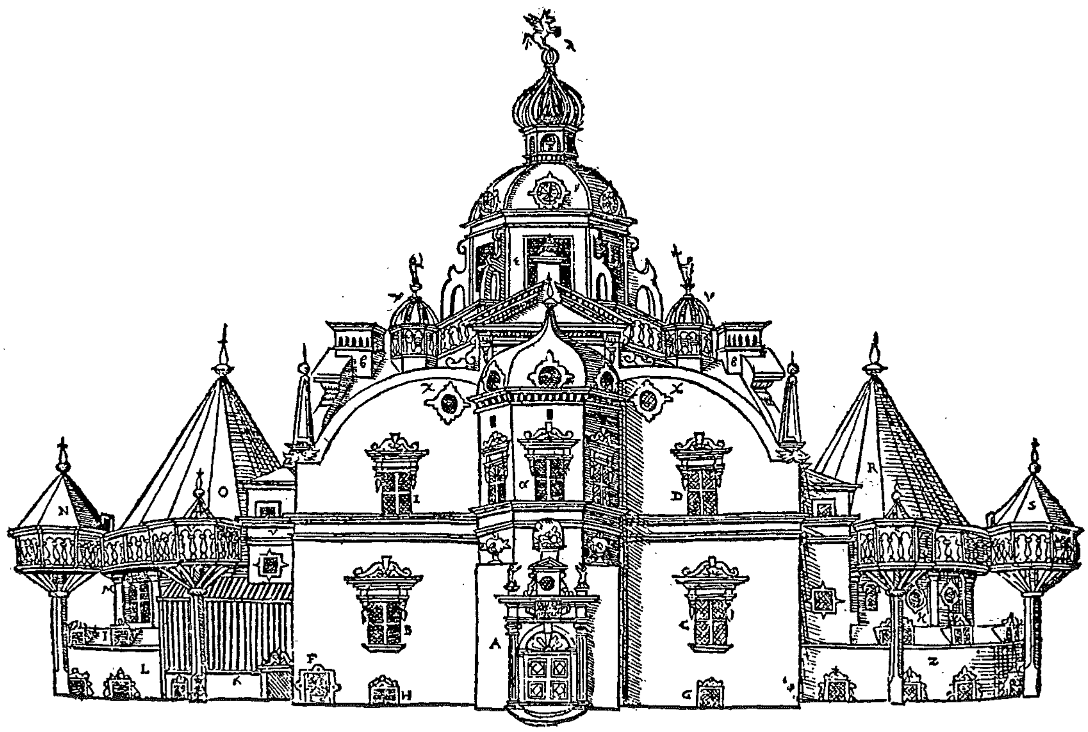

Figure 1. The observatory of Uranienborg (Brahe, 1596).

Figure 2. The large quadrant in Uranienborg. The persons in the foreground are observers; the area above the quadrant itself is covered by a wall painting. (Brahe, 1598.)

the stars could be determined. And once the declinations of the stars were known it would be possible to find the latitudes of arbitrary places as a basis for mapping and navigation.

Brahe's latitude measurements at Uranienborg were of an unprecedented accuracy. We will here compare his result, not only with a few later ones, but primarily with the result that can be obtained by combining modern satellite positioning and a gravimetric determination of the deflection of the vertical. The deflection of the vertical is necessary for transforming a satellite-derived latitude to a star-derived latitude, the former being referred to the Earth ellipsoid and the latter being referred to the geoid. In addition we will comment on the special pattern of the deflections of the vertical in the area.

2. Brahe's latitude determination of 1596

Brahe determined the latitude of Uranienborg through observing circumpolar stars. Such a star will transit the meridian twice a day, once on the upper side of the pole, upper culmination, and once on the lower side, lower culmination. We denote the altitude of the star at upper culmination by h_u and at lower culmination by h_l. Then the altitude of the pole can be identified as the mean value of the two observed altitudes of the star, and this is equal to the latitude Φ of the observation point:

$$\Phi = \frac{h_u + h_l}{2} \qquad (1)$$

Brahe measured the two meridian altitudes of the pole star as well as other circumpolar stars to determine the latitude of Uranienborg. He continued with this work through all the 20 years he spent there. Finally Brahe (1596) published a value of the latitude, Φ = 55°54′30″. This value is not stated in the text but on a small map of Ven with Uranienborg shown at the mentioned latitude, probably the first time seconds of arc are used in this context. Soon after that, Brahe (1598) published the same map again, now stating explicitly in the text that the latitude of Uranienborg amounts to 55°54½′. According to Picard (1680), the later and thereby best part of Brahe's measurements resulted in a latitude of Φ = 55°54′40″.

However, Brahe did not apply any correction for refraction of the star light in the atmosphere, because he had arrived at the conclusion that refraction was insignificant for altitudes above 45°. This is not quite the case. The effect of refraction on the altitude of a star and, thereby, on the latitude of the point on the Earth, may be calculated as r = 58.2″ cot h, for h > 30° (see e.g. Smart, 1962). Inserting h = 55.9° gives r = 40″. Subtracting this from Brahe's latitude values we obtain his latitudes corrected for refraction:

Φ = 55°53′50″ (all measurements)
Φ = 55°54′00″ (best measurements)

3. Picard's latitude determinations of 1680

Nearly 100 years after the foundation of Brahe's observatory at Uranienborg an astronomical observatory was founded in Paris. Here Picard introduced the use of telescopes when making astronomical and geodetic measurements. One of the first tasks was to determine the positions of Paris and Uranienborg relative to each other. For this purpose Picard travelled to the island of Ven, in 1671. Arriving there he realized that the observatory had been demolished into ruins by order of the Danish king after Tycho Brahe had left it, and the island itself had recently been ceded by Denmark to Sweden.

Picard, together with his Danish colleague Rømer, now determined the latitude of the ruins of Uranienborg. This was made in two different ways, published by Picard (1680). First, the latitude was determined using a circumpolar star, through (1), resulting in Φ = 55°55′20″, after correction for refraction Φ = 55°54′40″. Second, the latitude was determined using a star close to zenith, where refraction is (almost) zero, through

$$\Phi = \delta + 90^\circ - h \qquad (2)$$

δ being the declination of the star, in this case obtained at the Paris observatory. This resulted in Φ = 55°54′15″. We summarize Picard's latitude values (corrected for refraction):

Φ = 55°54′40″ (circumpolar observations)
Φ = 55°54′15″ (zenith observations)

Both values are slightly larger than those of Brahe. Later on, in 1765, the last-mentioned of Picard's values was adopted as a starting value in the Swedish coastal triangulation around the Baltic Sea (Schenmark, 1765).

4. Later latitude determinations

During the following centuries Uranienborg was included in the various triangulations of Sweden as an ordinary triangulation point. The latitude values obtained in that way are not comparable with the astronomical ones above because of the deflection of the vertical.

A renewed astronomical determination of the latitude of Uranienborg was performed in conjunction with the Danish work for connecting the triangulation networks of Denmark and Sweden, by Madsen & Sand (1908). Their result, based on (2), was

Φ = 55°54′24.5″

(with an internal standard error less than 0.1″). In addition they performed a local triangulation on Ven which will be useful in the following section.

5. The latitude from satellite positioning

The geocentric coordinates of a first order triangulation station on the south coast of Ven has been determined using modern satellite positioning (GPS). This determination was performed within a national campaign for establishing GPS coordinates on old triangulation stations. The resultant coordinates are obtained in the Swedish reference system SWEREF 99 (Jivall, 2001), in this context more or less identical to the European system EUREF 89 as well as the global systems ITRF 89 and WGS 84. The geocentric coordinates can then be transferred into latitude, longitude and height relative to the Earth ellipsoid (GRS 1980). The latitude obtained in this way for the GPS station Ven is $\varphi_V = 55°53'32.04''$.

The ruins of the observatory of Uranienborg are situated in the middle of the island of Ven, 2 km north of the GPS station. The latitude difference between the observatory and the GPS station, the latter being identical to the triangulation station, has been determined already in connection with the local triangulation there by Madsen & Sand (1908). The difference was found to be $\Delta\varphi = 56.08''$. Adding this quantity to the latitude of the GPS station we find the latitude in SWEREF 99 of the observatory:

$$\varphi = 55°54'28.12''$$

This latitude refers to the estimated location of Brahe's large wall-mounted quadrant. The location has been possible to identify through the detailed descriptions of the observatory buildings published by Brahe (1598).

The satellite-derived latitude thus found is not directly comparable with the star-derived latitudes of the earlier sections. The difference is due to the deflection of the vertical which we now need to find.

6. The gravimetric deflection of the vertical

Determining a latitude by astronomical positioning means measuring vertical angles towards a star. When putting up the instrument for measuring angles it is adjusted with a spirit level. The spirit level "feels" the direction of the plumb line, or the vertical. The vertical, being the normal to the geoid, deviates from the normal to the ellipsoid. This deviation, known as the deflection of the vertical, directly affects the astronomically determined latitude.

Determining a latitude by satellite positioning means measuring distances through timekeeping of radio waves emitted from the satellites. This procedure is independent of any spirit level and, hence, does not depend on the direction of the vertical. Thus the latitude so determined is unaffected by the deflection of the vertical.

Denoting the star-derived or astronomical latitude by Φ and the satellite-derived or geodetic latitude by φ we may write

$$\Phi = \varphi + \xi \tag{3}$$

Here ξ is the deflection of the vertical in the south-north direction.

Now, the deflection of the vertical at a certain point is nothing but the inclination of the geoid relative to the ellipsoid at that point. Thus the deflection of the vertical ξ can be computed as the derivative of the geoid height N in the south-north direction,

$$\xi = -\frac{\partial N}{R\,\partial\varphi} \tag{4}$$

R being the mean radius of the Earth.

The geoid and, thereby, the deflections of the vertical are due to the irregular mass distribution within the Earth. Hence the geoid can be computed from a detailed and global knowledge of the Earth's gravity field. Such a knowledge has only been achieved during the last decades. Modern geoid computations are based on a combination of satellite orbit perturbations, surface gravity anomalies, and digital terrain models. The most recent global geoid model is EGM 2008 of Pavlis et al (2008). According to that we obtain at Uranienborg

$$\xi = -3.6''$$

This is thus the deflection of the vertical in latitude at Brahe's observatory. We have also used the recent geoid model SWEN08_RH2000 over Sweden by Ågren (2009). This yields nearly the same result,

$$\xi = -3.3''$$

According to our judgement in this specific case the two values should be equally reliable.

Now we are in the position to apply (3) to find the astronomical latitude of Uranienborg. Combining the satellite-derived latitude in the previous section and the two gravimetric deflections of the vertical above, we obtain

$$\Phi = 55°54'24.5''$$

in the first case and

$$\Phi = 55°54'24.8''$$

in the second case.

7. Final results and conclusions

For the ruins of Tycho Brahe's observatory of Uranienborg we have found, using satellite positioning (GPS) and gravimetry, values of the geodetic latitude, the deflection of the vertical, and the astronomical latitude. These are summarized in the first and second columns below. In addition we have found, in the third column, values of the same quantities using GPS and the "modern" astronomical positioning (a century ago) of Madsen & Sand in Section 4.

GPS & grav. [1]	GPS & grav. [2]	GPS & astr.
$\varphi = 55°54'28.1''$	$\varphi = 55°54'28.1''$	$\varphi = 55°54'28.1''$
$\xi = -3.6''$	$\xi = -3.3''$	$\xi = -3.6''$
$\Phi = 55°54'24.5''$	$\Phi = 55°54'24.8''$	$\Phi = 55°54'24.5''$

The geodetic latitude in the three columns can be considered error-free. The uncertainty in the deflections of the vertical in the first two columns can be estimated at 0.2", yielding the same uncertainty for the astronomical latitude there. The uncertainty in the astronomical latitude in the third column can be estimated at some 0.2" (yielding the same uncertainty for the deflection of the vertical there). Thus the astronomical latitude obtained from GPS and gravimetry agrees, within the small uncertainties, with the "modern" astronomical positioning. This shows an excellent consistency between satellite positioning, the gravimetric determination of the deflection of the vertical, and astronomical positioning.

The obtained value of the deflection of the vertical is quite interesting. The geoid in general over the Nordic area is tilted in the west-east direction (see e.g. Forsberg et al, 1996). This should yield fairly large deflections of the vertical (up to about 10") in that direction, but close to zero in the south-north direction. At Uranienborg the situation is found to be the opposite, the deflection of the vertical being 4″ in the south-north direction but close to zero (1″) in the west-east direction. This can be put into a wider context by studying the geoid over the surrounding Danish-Swedish area; see Figure 3. Here a zone running from north-west to south-east is clearly evident, along which the geoid heights are "displaced", causing a sharply bended shape of the curves. This zone is identical to the so-called Tornquist zone, a probable remnant of an old plate boundary. It is characterized by a shallow density contrast reflected in a very steep gradient in the Bouguer gravity anomaly (SGU, 1997). Across the zone Kind et al (1997) have also found an abrupt change in Moho depth, i.e. crustal thickness. Uranienborg happens to be situated in this special zone, causing the obtained deflection of the vertical in latitude.

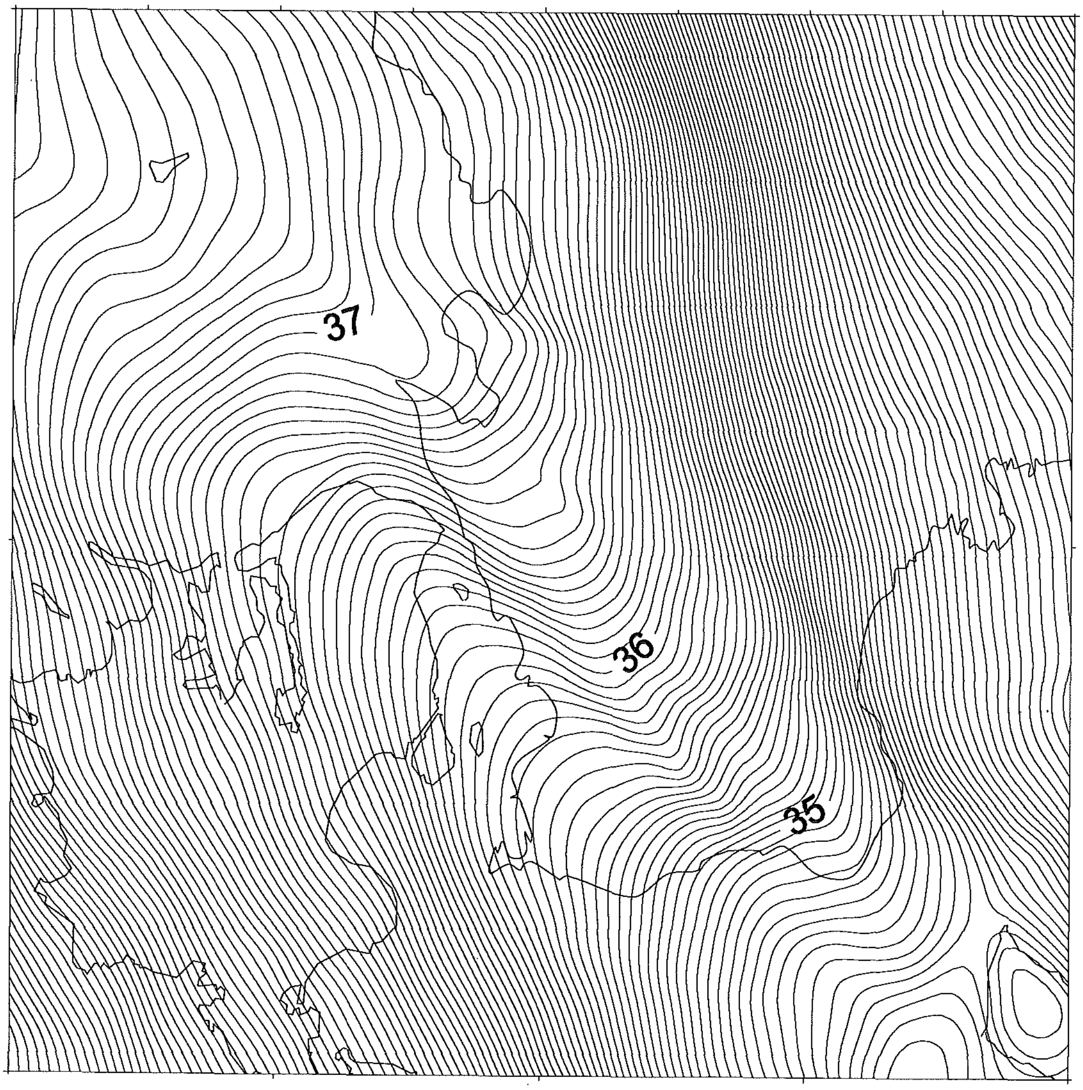

Figure 3. Map of geoid heights (m) over eastern Denmark and southern Sweden (based on Ågren, 2009). Uranienborg is situated on the small island in the centre of the figure.

Let us now compare our results above, rounded to whole seconds, with the astronomical latitude of Brahe (best measurements) in Section 2, and also with the two values of Picard in Section 3.

Above	Brahe	Picard
φ = 55°54′28″		
ξ = - 4″		
Φ = 55°54′24″	Φ = 55°54′00″	Φ = 55°54′40″; 55°54′15″

We find an error in Brahe's latitude determination of only 24″. This is a remarkable achievement bearing in mind that he did not have a telescope. A corresponding comparison with Picard's telescopic values a century later yields errors of 16″ and 9″, respectively, not too much better than Brahe's result. Finally we may note that the deflection of the vertical of 4″ is not insignificant in relation to the errors in Brahe's and Picard's latitude determinations.

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ISSN 1798-1883 (print)
ISSN 1798-1891 (online)