Small Publications in Historical Geophysics

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Gravity Determinations at the Observatories of Uppsala and Stockholm during three Centuries

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Summer Institute for Historical Geophysics
Åland Islands

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1. Introduction

Gravity has been possible to measure in different places on the Earth since the early 1700s. In the beginning, and for a long time, gravity was determined using swinging pendulums. Later elastic springs also came into use for finding gravity differences. Today gravity can be accurately determined using falling bodies. The uncertainty in the gravity determinations has decreased by several orders of magnitude during these centuries; see Torge (1989).

Gravity determinations were initiated as a means to study the rotational flattening of the Earth. Decreasing uncertainty in the gravity determinations later allowed studies of the irregular mass distribution within the Earth. Today also changes in the mass distribution due to various geodynamic phenomena are possible to detect through repeated gravity determinations.

Early gravity determinations were made in London and Paris, and soon also at the Arctic Circle in northern Scandinavia in connection with the French arc measurement expedition there led by Maupertuis. As Celsius from Sweden participated in this event he was inspired to make an early gravity determination also at his observatory in Uppsala. The two sister observatories of Uppsala and Stockholm now have a series of half a dozen gravity determinations made with different methods at various times during three centuries, including a modern one. These data will be used to illustrate the decrease of errors in gravity determination, absolute determination as well as relative, during this time span. Moreover, the data will be used for comparing the true errors with the standard errors estimated at the time of measurement.

2. Gravity measurements

In the beginning gravity was always determined absolutely, using the principle of a swinging pendulum. As is well known, the period (swinging time) of a pendulum is dependent on the value of gravity. Already in 1741 Celsius had a special pendulum clock installed at the Uppsala Observatory for determining the value of gravity there; see Celsius (1744). This pendulum clock was constructed for him by Graham in London. It was especially designed to be less sensitive to temperature changes influencing the length of the pendulum.

An improved pendulum, the reversible pendulum, was developed by Kater in London in the early 1800s. Soon after that such a pendulum was installed at the Stockholm Observatory by Svanberg (1825) for determining gravity there. An additional purpose of this was to fix more accurately the definitions of length, time and mass. After some years the gravity determination was repeated, now applying some further improvements; see Svanberg (1834). He himself considered the latter determination superior.

Towards the end of the 1800s a new kind of gravity apparatus was constructed by Sterneck in Vienna. This also used the pendulum principle, but it was a portable instrument designed for measuring differences in gravity. This made absolute pendulum gravity measurements necessary only in a few places; other gravity stations could now be more easily determined with relative pendulum measurements. An absolute gravity station was established in Vienna and a little later one in München; from these, gravity was transferred through special relative measurements to the Geodetic Institute at Potsdam outside Berlin as stated by Sterneck (1891) and Borrass (1896). Potsdam from then on served as a fundamental gravity station. Soon after that gravity was determined at the Uppsala and Stockholm Observatories through relative measurements from Potsdam. These were performed by Rosén (1898) using the Sterneck pendulum instrument. The connection to Potsdam was made via Lund, and some stations to the north of Uppsala were also included.

In the early 1900s a thorough absolute determination of gravity was made at Potsdam by Kühnen & Furtwängler (1906), leading to a revised fundamental gravity value. Based on this, relative gravity measurements were later on made with a new kind of relative gravimeter. This used the principle of an elastic spring as developed by Ising in Stockholm, Nørgaard in Copenhagen and others. In this way, within the framework of the Baltic Geodetic Commission and a gravity network over Sweden in the 1940s, the Uppsala Observatory was again connected to Potsdam, this time via Helsinki and Stockholm. These connections to Potsdam were made by Schmehl & Andersen (1937) and Wideland (1942, 1946), using an improved Sterneck pendulum apparatus as well as a Nørgaard spring gravimeter.

Already at this time it was suspected that the Potsdam value of gravity was in error by an amount considerably larger than its estimated uncertainty. An unofficial revised Potsdam value of gravity, partly based on a recomputed absolute determination at Potsdam, partly on an absolute determination in London by Clark (1939), was presented in the middle of the 1900s; see Berroth (1949). However, the old official Potsdam value continued to be used in parallel for several decades, until further experiments could confirm the need for revision. These further experiments were based on a new principle now introduced, that of timing freely falling bodies, by Cook (1965, 1967). At about the same time, within the framework of the gravimetric European Calibration Line and a new gravity network over Sweden in the 1960s, the Uppsala Observatory was once again connected to Potsdam, this time via Copenhagen, Oslo and Stockholm. These connections to Potsdam were made by Kneissl &

Marzahn (1963) and Pettersson (1967), now using mainly Worden spring gravimeters. The Stockholm Observatory was no longer in use and was, therefore, no longer measured, but two points close to it were, allowing interpolation to obtain gravity there.

In the 1970s absolute gravity became possible to determine using a transportable absolute gravity apparatus based on the free-fall principle, first developed by Faller in Boulder. In the Nordic countries such an absolute gravity station, observed several times, was established at the small Geodetic Observatory at Mårtsbo outside Gävle, somewhat north of Uppsala. The absolute measurements there were made by Cannizzo et al (1978) and recomputed by Haller & Ekman (1988) for a new fundamental gravity network in Sweden. The connection between the Uppsala Observatory and this absolute station was made with LaCoste & Romberg spring gravimeters, mainly by Haller & Ekman (1988) and Engfeldt (2016).

Since then further developments have led to the construction of portable free-fall absolute gravimeters of high accuracy. Such absolute measurements have now been performed at Mårtsbo with a number of instruments during several years; see Olsson et al (2017). The authors of the present publication have, therefore, determined gravity once again at the Uppsala Observatory, this time by connecting it to the redetermined value of the Mårtsbo absolute gravity station. The connection has been made using Scintrex spring gravimeters.

An overview of the principle methods used for all the gravity determinations is given in Table 1.

Table 1. Gravity determinations at the Observatories of Uppsala and/or Stockholm 1744 – 2017, principle methods.

Reference	Method
Celsius (1744)	Abs. (pendulum)
Svanberg (1825, 1834)	Abs. (rev. pendulum)
Rosén (1898)	Abs. (rev. pendulum) + rel. (pendulum)
Wideland (1942, 1946)	Abs. (rev. pendulum) + rel. (pendulum / spring)
Pettersson (1967)	Abs. (rev. pendulum / fall) + rel. (spring)
Haller & Ekman (1988) a.o.	Abs. (fall) + rel. (spring)
This publ. (2017)	Abs. (port. fall) + rel. (spring)

3. Gravity values

The resultant values of gravity at the Observatories of Uppsala and Stockholm through the centuries are presented in Tables 2a and 2b. We will here explain how these values have been extracted and, in some cases, modified from the original publications.

Table 2a. Gravity determinations at the Uppsala Observatory 1744 – 2017, results. Unit: m/s^2.

Reference	Gravity
Celsius (1744)	9.81 54
Rosén (1898)	9.81 924
Wideland (1946)	9.81 899 4
Pettersson (1967)	9.81 885 85
Haller & Ekman (1988) a.o.	9.81 883 817
This publ. (2017)	9.81 883 801

Table 2b. Gravity determinations at the Stockholm Observatory 1825 – 1967, results. Unit: m/s^2. (Asterisk denotes interpolation.)

Reference	Gravity
Svanberg (1825)	9.81 865
Svanberg (1834)	9.81 821
Rosén (1898)	9.81 858
Pettersson (1967)*	9.81 825 9
Modern value (calculated)	9.81 823 9

The gravity result obtained by Celsius at Uppsala is taken from Celsius (1744). A transformation has been made from the length of the pendulum swinging seconds, as given in Swedish feet and decimal inches by him, into gravity expressed in m/s^2. His determination is not made in the same observatory building as the later ones in Uppsala, but that is no problem since the gravity difference between the buildings is less than one unit in the last digit.

The two gravity results obtained by Svanberg at Stockholm are taken from Svanberg (1825, 1834). Transformations from the length of the pendulum to gravity in the modern sense have been made also here. His published values refer to sea level as reduced by a Bouguer reduction; we have reduced this back again to refer to the actual site. Svanberg himself considered his later measurement superior to the earlier one; the standard errors in Table 3b (next section) clearly support this view. Hence we adopt the later determination as the main result (a weighted mean would in any case be equal to this).

The results of Rosén at both Uppsala and Stockholm are taken directly from Rosén (1898). He has used Borrass (1896) for the value at Potsdam. (It should be noticed that Rosén, when commenting on Svanberg's values, does not seem to have realized that his values were reduced to sea level.)

The result of Wideland at Uppsala is taken from Wideland (1942, 1946), He has (partly) used Schmehl & Andersen (1937) for the difference between Helsinki and Potsdam, and Kühnen & Furtwängler (1906) for the absolute Potsdam value.

The result of Pettersson at Uppsala is taken from Pettersson (1967). He has used Kneissl & Marzahn (1963) for the difference between Oslo/Copenhagen and Potsdam. For Potsdam an unofficial revised gravity value based on the recomputed absolute value there by Berroth (1949), together with the absolute determinations in London by Clark (1939) and Cook (1965, 1967), has been applied. The published value for Uppsala was based on the official value at Potsdam, but since an unofficial value like this one was used in parallel at that time for scientific purposes we have applied it here. (The complete abandoning of Potsdam came with Morelli (1974).) The corresponding Stockholm value is, as mentioned, obtained through interpolation only, between two nearby points (using Bouguer anomalies).

The next result for Uppsala is founded on Haller & Ekman (1988) with some additional data from Engfeldt (2016). Their absolute value at Mårtsbo, based on Cannizzo et al (1978), refers to the year 1982; we have reduced it for ongoing postglacial rebound to the year 2000 to make it comparable with the present result below. (Also, we have applied the same vertical gravity gradient as for the absolute determination below.)

The result of the present authors for Uppsala needs the following comments. The accuracy of modern gravity determinations requires them to be corrected for perturbing geophysical effects. In particular, geodynamic phenomena like the permanent tide and the postglacial rebound influence gravity in a systematic way, making it necessary to specify how these phenomena are treated in order to define the gravity concept uniquely. Without going into details we make it clear that gravity here with regard to the permanent tide is zero tide gravity (Ekman, 1989, 1996), and that it with regard to the postglacial rebound refers to the year 2000 (Mäkinen et al, 2004; Olsson et al, 2015). The absolute gravity at Mårtsbo, defined in this way, is based on the results of Olsson et al (2017). The gravity difference between this main absolute station and Uppsala is the average of the results from two instruments travelling forth and back.

The latest gravity values of the Uppsala Observatory refer to the top level of the main entrance (southern part). Older values referring to other points have been safely reduced to this modern point.

4. Gravity errors

Table 3a. Gravity determinations at the Uppsala Observatory 1744 – 2017, errors. Unit: m/s^2.

	Error in absolute gravity		Error in relative gravity	
Reference	True	Standard	True	Standard
Celsius (1744)	.00 35	(.00 20)	-	-
Rosén (1898)	.00 032	.00 010	.00 008	.00 007
Wideland (1946)	.00 014	.00 003	.00 001 6	.00 001 5
Pettersson (1967)	.00 001 4	.00 001 0	.00 000 6	.00 000 2
Haller & E (1988)	.00 000 019	.00 000 008	.00 000 003	.00 000 015
This publ. (2017)	Estimated values see Table 4			

Table 3b. Gravity determinations at the Stockholm Observatory 1825 – 1967, errors. Unit: m/s^2.

	Error in absolute gravity		Error in relative gravity	
Reference	True	Standard	True	Standard
Svanberg (1825)	.00 041	.00 070	-	-
Svanberg (1834)	.00 003	.00 020	-	-
Rosén (1898)	.00 032	.00 010	.00 002	.00 007
Pettersson (1967)	.00 001 4	.00 001 0	.00 000 6	.00 000 3

The true errors in all the gravity determinations of the foregoing section have now been calculated on the basis that our modern value is "correct", at least in relation to the preceding ones. The true errors thus obtained are presented in Tables 3a and 3b. Each error is, where relevant, divided into the error in the absolute determination and the error in the relative determination.

For comparisons also the standard errors estimated at the time of the measurements are shown. Most of these standard errors are taken from, or deduced from, the publications used for the gravity values in Section 3. However, the standard errors for Svanberg´s determinations have been estimated by the authors from data given in his papers. The standard error within brackets for Celsius' determination is a guessed one on the basis of information in Ekman & Mäkinen (1998) and Ekman (2016).

In Table 4 we have merged the above tables into a chronological overview of the errors, the true errors as well as the standard ones. Both kinds of errors are given separately for absolute and relative determinations. This overview should illustrate the general error development in gravity as well as differences between standard errors and true errors.

Table 4. Gravity determinations at the Observatories of Uppsala and/or Stockholm 1744 – 2017, error overview. Unit: m/s^2.

	Error in absolute gravity		Error in relative gravity	
Reference	True	Standard	True	Standard
Celsius (1744)	.00 35	(.00 20)	-	-
Svanberg (1834)	.00 003	.00 020	-	-
Rosén (1898)	.00 032	.00 010	.00 008	.00 007
Wideland (1946)	.00 014	.00 003	.00 001 6	.00 001 5
Pettersson (1967)	.00 001 4	.00 001 0	.00 000 6	.00 000 2
Haller & E (1988)	.00 000 019	.00 000 008	.00 000 003	.00 000 015
This publ. (2017)		.00 000 002		.00 000 005

Let us now study the general development in Table 4. When absolute gravity measurements starts in the first half of the 1700s with the use of the pendulum, Celsius (1744) achieves an accuracy of the order of 100 mgal, where 1 mgal = 10^{-5} m/s^2. This is in accordance with the very few other gravity determinations in the world at that time; see Ekman & Mäkinen (1998) and Ekman (2016).

In the early 1800s Svanberg (1834) succeeds in increasing the accuracy of absolute gravity to the order of 10 mgal, one order of magnitude better than before. This is due to the introduction of the reversible pendulum at this time.

Towards the end of the 1800s the accuracy of the absolute gravity has not improved. What happens instead is that relative gravity has become possible to determine more easily with the pendulum instrument. Rosén (1898) in this way achieves a relative accuracy of the same order, 10 mgal, as the absolute one.

Before the middle of the 1900s the accuracy of the absolute gravity has still not improved. However, with Wideland (1946) the relative accuracy is increased to the order of 1 mgal, one order of magnitude better than before. This is due to both an improved pendulum instrument and the introduction of the new spring gravimeter at this time. Thereby relative gravity becomes one order of magnitude better than absolute gravity (1 mgal versus 10 mgal).

In the early second half of the 1900s, after a stand-still for more than 100 years, the accuracy of the absolute gravity is improved by one order of magnitude, to 1 mgal. This is partly due to an improved reversible pendulum but mostly to the introduction of the new free-fall method. At the same time, with Pettersson (1967), also the relative accuracy is increased somewhat by using an improved spring gravimeter, but the improvement is less than one order of magnitude. Hence the gap between relative and absolute gravity now begins to close again.

In the late second half of the 1900s the values according to Haller & Ekman (1988) show that the absolute as well as the relative gravity has reached an accuracy of 0.01 mgal. Thus the accuracy in gravity has improved by two orders of magnitude in a few decades, especially due to improved free-fall instruments but also to improved spring gravimeters. In addition, corrections for geodynamic phenomena were now introduced, particularly for earth tides and postglacial rebound. Moreover, the use of a transportable absolute gravimeter made it possible to again determine absolute gravity "at home".

Finally, as illustrated by the recent determinations, the absolute as well as relative gravity now tend to approach an accuracy of the order of 0.001 mgal; cf. also Niebauer (1995) and Van Camp et al (2005). Furthermore, the use of portable absolute gravimeters has made it possible to determine absolute gravity with several different instruments, and also at several different sites. On the other hand, the accuracy achieved means that one also approaches the limit where the result becomes significantly influenced by minor local effects.

We now turn to the relation between the estimated standard errors and the true errors in Table 4. In several cases the two kinds of errors agree reasonably. In some cases, however, we note significant differences. First we find a surprisingly small true error in Svanberg's (1834) value, of the order of one tenth of his standard error and not surpassed until more than 100 years later. This may be interpreted as a combination of good luck and skill. We know of one similar success at that time, the gravity determination by Bessel (1835) at the Berlin Observatory.

On the other hand, we have some examples of what is known as a fairly common problem: The true error turns out to be significantly larger than the estimated standard error. This is especially the case, as is already known, with the absolute Potsdam value of Kühnen & Furtwängler (1906), applied by Wideland (1946). The discrepancy (five times the standard error), is explained by Berroth (1949) as a combination of certain systematic errors of a technical nature. As regards relative gravity, Pettersson (1967), based on Kneissl & Marzahn (1963), shows a true error significantly larger than the standard error. This reflects a scale error in the relative determinations.

5. Conclusions

Table 5. Orders of magnitude of uncertainties in gravity (in m/s^2) during three centuries.

Century	Absolute gravity	Relative gravity
1700s	10^{-3}	
1800s	10^{-4}	
1900s, 1st half	10^{-4}	10^{-4}
1900s, middle	10^{-4}	10^{-5}
1900s, 2nd half, early	10^{-5}	10^{-5}
1900s, 2nd half, late	10^{-7}	10^{-7}
2000	10^{-8}	10^{-7} - 10^{-8}

The series of gravity determinations at the Uppsala and Stockholm Observatories from 1741 up till today reflects the major steps in the development of gravity observations. These include the absolute swinging pendulum, the absolute reversible pendulum, the relative pendulum apparatus, the relative spring gravimeters, and the recent absolute free-fall methods. This has required improved methods of gravity computations, too, including corrections for geodynamic effects.

The decreasing errors in the gravity values of the Uppsala and Stockholm Observatories are shown in Table 4. This reveals the way the errors have decreased in the absolute sense as well as the relative sense. To further show the general development through the centuries we have summarized our gravity errors in terms of orders of magnitude in Table 5. In total, gravity has improved by no less than 5 orders of magnitude in 300 years, from an uncertainty of 10^{-3} m/s^2 to 10^{-8} m/s^2, most of it during the last century. Absolute gravity, introduced in the middle of the 1700s, improved by one order of magnitude in the beginning of the 1800s, but did then come to a stand-still for 100 years, until it suddenly improved by 4 orders of magnitude in the latter half of the 1900s. Relative gravity, introduced in the late 1800s, has improved more gradually, by 3 – 4 orders of magnitude since then. In general, the rapid development during the last century is due to replacing pendulum methods by spring and free-fall methods.

The relation between true errors and estimated standard errors is also shown in Table 4. In general they agree reasonably, but there are some examples of too “optimistic” standard errors, and one early example of a true error much smaller than expected.

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