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An Investigation of a Pioneering Triangulation across the Åland Islands

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Summer Institute for Historical Geophysics
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1. The French background

In the 1730s an international scientific controversy with wide implications arose concerning the shape of the Earth. In England Newton had, based on his theories of gravitational and centrifugal forces, arrived at the conclusion that the Earth must be a body somewhat flattened at the poles. In France Cassini at the Paris observatory had, based on his geodetic measurements across the country, arrived at the conclusion that the Earth must be a body somewhat flattened at the equator, thus contradicting Newton's theories. In order to solve the problem the French Academy of Sciences decided to organize two scientific expeditions, one to the south, close to the equator, and one to the north, as far north as possible.

Following a proposal from Celsius, at that time visiting France, the northern expedition was sent to northern Sweden (now Sweden and Finland), more specifically to the area of Torneå (Tornio) at the end of the Gulf of Bothnia, close to the Arctic circle. The expedition was headed by Maupertuis, with Celsius as the only non-French member. The main task for Maupertuis' expedition was to perform a meridian arc measurement, i.e. to determine the distance as well as the latitude difference between the end points of a meridian arc. A comparison of such a result in the north with a corresponding result from an arc in the south, in France or at the equator, would give information on the flattening of the Earth.

The distance between the end points was far too long to be measured directly, but had to be found using a special method, triangulation. Triangulation had at that time recently been developed into a useful method by extensive works in France. Knowing the distance as well as the latitude difference of the meridian arc, its curvature could be computed.

Now, comparing the result of the northern expedition with a corresponding result in France, and later on with the result from the equatorial expedition, Maupertuis et al (1738) found that the meridional curvature of the Earth is smaller closer to the pole and larger closer to the equator. From this they concluded that the Earth is flattened at the poles. Their conclusion was confirmed by the gravity measurements made by the expedition, analyzed by Clairaut (1743). Although the accuracy of Maupertuis' result later turned out to be clearly less than he himself had imagined, the total result of the expedition supported the theories of Newton and, in any case, contradicted the findings of Cassini.

The result of Maupertuis' expedition to the north not only had scientific but also practical consequences. In France triangulation had been applied by

Cassini in order to construct a foundation for an accurate map of the whole country. The results from the north now brought about a partial remeasurement of the French triangulation, using one of the expedition's two angle instruments, to improve the foundation of the map. The triangulation network of France was completed and published by Cassini's son and successor at the Paris observatory, Cassini de Thury (1744). This was the first triangulation in the world for constructing a national land map; it marked the beginning of a global revolution in the determination of coordinates for mapping the Earth.

The other (and somewhat smaller) of the two angle instruments of Maupertuis' expedition was used in another triangulation for accurate mapping: a triangulation across the Baltic Sea between Sweden and Finland, via the Åland Islands. This appears to have been the first triangulation in the world for marine purposes, resulting in nautical charts as well as a land map.

2. The triangulation across the Åland Islands

The Åland Islands in the middle of the Baltic Sea between Sweden and Finland include an extensive archipelago comprising thousands of islands and skerries. Between and across these islands ran the important "Post route", used for transporting not only mail and goods but also diplomats and other people travelling between the western and eastern parts of northern Europe. This area certainly was in need of a more reliable mapping based on the new method.

The triangulation here across the Baltic Sea was organized as a cooperation between the Royal Survey Office of Sweden, the University of Åbo and, in the background, the University of Uppsala. The triangulation was carried out between 1748 and 1752 (or possibly 1754) under the leadership of Gadolin, a former student of Celsius. The triangulation network covered 3 ½ ° in longitude, approximately along the latitude 60°, from Väddö in eastern Sweden across the Åland Islands to Åbo (Turku) in south-western Finland; see Figure 1. The purpose of the triangulation was to determine the coordinates (latitude and longitude) for a number of stations as the basis for mapping and charting. The procedure for this triangulation may be described as follows; for details see Gadolin (1753, 1757).

The first step in a triangulation is to make an astronomical determination of latitude and longitude of one station. This is to position the triangulation network on the Earth. The astronomical station selected in our case was Åbo cathedral.

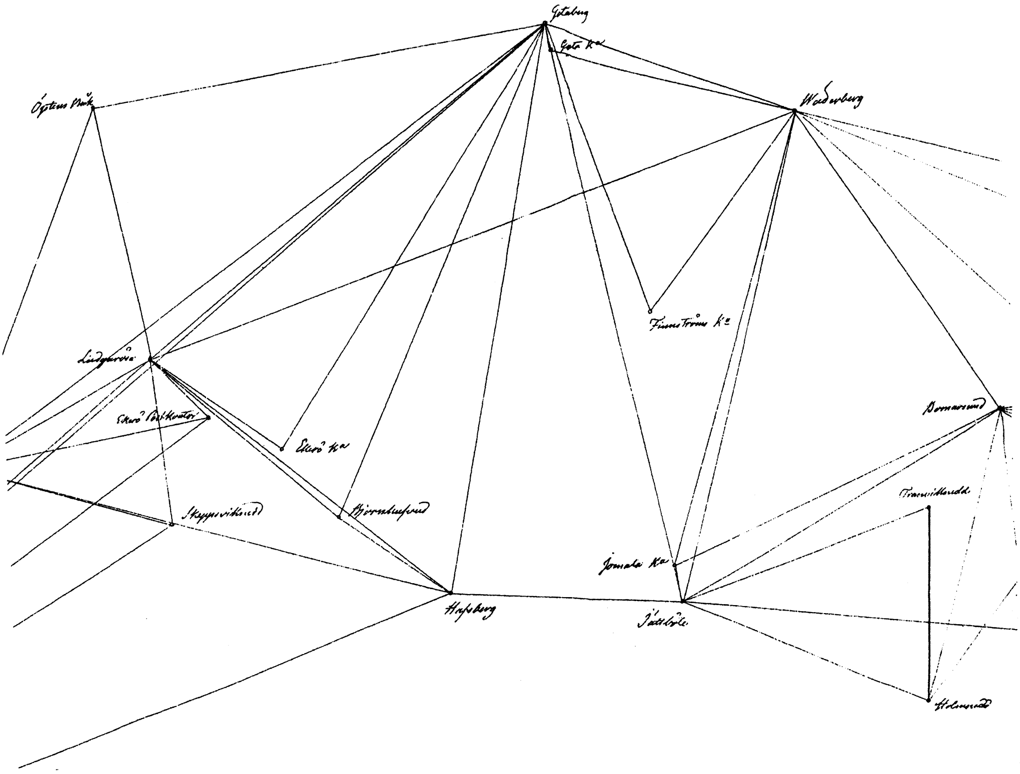

Figure 1. The central part of the triangulation net across Åland established around 1750. (From Hällström, 1815.)

The second step is to make an astronomical determination of an azimuth, the azimuth being the horizontal angle between the direction to the north and one of the sides in the triangulation network. This is to orientate the triangulation network on the Earth. In this case the azimuth of a triangle side along the west coast of Åland (Signilskär - Högsten) was determined. (Another azimuth further to the east in the network was measured with less accuracy as a check.)

The third step, and the most time-consuming work, is to measure all horizontal angles between the triangulation stations in the network. To make such measurements one first has to select suitable stations with good sights towards other stations, and on each station erect some kind of signal to be measured against from the neighbour stations. In our case of the Åland Islands the triangulation stations could be selected in a very special way, namely among the beacon cliffs there. They constituted an ancient warning system in case of threatening attacks. Not only did these cliffs have excellent sights between them, but they were also already monumented with signals in the form of wooden beacons to be set fire on in case of emergency. In addition some churches with towers as well as skerries with nautical constructions could be used for triangulation. In total the triangulation network comprised some 40 stations, with sight lines of up to 40 km (in one case 70 km).

The fourth step is to measure the length of a baseline. When selecting a suitable baseline one should look for a reasonably flat surface close to sea level. In the case of the Åland Islands there appeared an ideal and almost unique possibility: The baseline could be measured directly on the ice of the sea. A 10 km distance was measured on a part of the ice-covered sea at Åland (Lumparn) surrounded by sheltering islands, thereby reducing possible movements of the sea ice. (A shorter baseline on land further to the east was measured as a check.)

The final step is to compute the coordinates of all triangulation stations. Starting from the latitude and longitude of the astronomical station, and applying trigonometry to the angles and the baseline in the net of triangles, the latitudes and longitudes of all triangulation stations can be found. The calculations are complicated by the curvature of the Earth, but we leave that aside here.

The resultant coordinates of the triangulation across the Baltic Sea and the Åland Islands were presented by Gadolin (1757). These coordinates, together with other ones from connected triangulations performed later, served as a foundation for the construction of the first accurate nautical chart in the Baltic Sea, produced by Nordenankar (1783). The coordinates were also used for constructing a special map, showing the Post route across the Åland Islands, produced by Wetterstedt (1789).

3. Comparisons of the results with modern coordinates

Let us now investigate the results of this historical triangulation. When doing so we utilize the recomputation of the whole triangulation executed by Hällström (1815), using improved mathematical methods. For the investigation 13 main stations are selected, reasonably distributed within the triangulation network and having long sight lines to several neighbour stations. These stations are listed in Table 1. The first three of them are situated in Finland, the following nine ones on Åland, and the last one in Sweden. For each station are given its measured and modern latitude, in the upper part of the table, and its

Table 1. Coordinates of the triangulation across the Baltic Sea via the Åland Islands in 1748 - 1752 (in degrees, minutes and seconds), and their errors (in seconds). Stations are ordered from east to west.

Station	Meas. lat.	Modern lat.	Error
Åbo (Turku) cathedral	60 27 06	60 27 09	- 3
Prostvik beacon cliff	60 12 43	60 12 43	0
Korpo church	60 09 47	60 09 47	0
Kumlinge beacon cliff	60 14 54	60 14 52	+ 2
Ulversböte beacon cliff	60 06 30	60 06 28	+ 2
Bomarsund beacon cliff	60 13 05	60 13 04	+ 1
Väderberg beacon cliff	60 20 47	60 20 48	- 1
Jomala church	60 09 21	60 09 18	+ 3
Getaberg beacon cliff	60 23 09	60 23 10	- 1
Havisberg beacon cliff	60 08 45	60 08 45	0
Högsten nautical beacon	60 21 15	60 21 13	+ 2
Signilskär nautical cairn	60 12 09	60 12 04	+ 5
Stacksten [Väddö] beacon cliff	59 57 57	59 57 57	(0)

Station	Meas. long.	Modern long.	Error
Åbo (Turku) cathedral	22 16 33	22 16 40	- 7
Prostvik beacon cliff	22 02 57	22 03 02	- 5
Korpo church	21 33 44	21 33 50	- 6
Kumlinge beacon cliff	20 47 24	20 47 05	+ 19
Ulversböte beacon cliff	20 33 52	20 33 46	+ 6
Bomarsund beacon cliff	20 13 50	20 13 40	+ 10
Väderberg beacon cliff	20 03 31	20 03 27	+ 4
Jomala church	19 56 59	19 56 59	0
Getaberg beacon cliff	19 50 33	19 50 40	- 7
Havisberg beacon cliff	19 45 22	19 45 19	+ 3
Högsten nautical beacon	19 27 30	19 27 13	+ 17
Signilskär nautical cairn	19 20 32	19 20 15	+ 17
Stacksten [Väddö] beacon cliff	18 50 18	18 50 18	(0)

measured and modern longitude, in the lower part of the table. These coordinates need some explanation before we enter into their analysis.

The astronomical starting point at the Åbo cathedral by necessity contains some error in the astronomical positioning. As we are not interested in that kind of error for investigating the accuracy of the triangulation we could eliminate it by assigning this station its modern coordinates, and then apply the same correction to all the other stations. In this way we also eliminate a principal difference between astronomical and modern coordinates due to deflections of the vertical. However, since the main part of the network has a rather weak connection to Åbo cathedral, it seems more relevant to assign the modern coordinates to the other end station, Väddö beacon cliff; this is a modern triangulation and satellite positioning station as well. Accordingly all the measured coordinates of the triangulation stations have been shifted by a constant in latitude (- 9″) and another constant in longitude (- 17°41′15″); the shift in longitude also contains a change of the zero meridian from Ferro to Greenwich. These translated coordinates are the ones given in Table 1 as measured coordinates.

The modern coordinates in Table 1 are given in a system based on satellite positioning, i.e. WGS 84 and its close relatives, in Finland EUREF-FIN and in Sweden SWEREF 99. All stations have been identified on present-day detailed maps and charts, and their coordinates taken from there; some of the stations are also modern triangulation stations. For the initial station, the western end station Väddö beacon cliff, the coordinates have been checked against the satellite positioning performed there. We should also mention that the effect of different reference ellipsoids being used for the measured and modern coordinates, respectively, is negligible in this case.

We now are in the position to study the errors of the measured coordinates. The error of a measured coordinate in Table 1 is calculated as the difference between measured and modern coordinates. Let us start with the latitudes. From their errors we find a standard deviation in latitude of

$$\sigma_\varphi = 2'' \approx 60\,\mathrm{m}$$

No systematic error can be found. Turning to the longitudes we notice some effects that do not appear quite random; they might be due to the design of the network with long sights and small angles in the longitudinal direction. Eliminating the small non-zero average of the errors (dependent on the choice of the initial station) we find a standard deviation in longitude of

$$\sigma_\lambda = 9'' \approx 140\,\mathrm{m}$$

(taking into account that the length of 1″ of longitude at the latitude 60° is one half of the length of 1″ of latitude). Thus the overall uncertainty of the coordinates of the triangulation may be said to be about 100 m, a surprisingly small quantity.

4. Some other comparisons

Gadolin himself tried to check his triangulation by making astronomical latitude determinations on seven of the triangulation stations. The discrepancies turn out to have a standard deviation of 8″ (although with a somewhat skew distribution). As the standard deviation of the triangulated latitudes is only 2″ according to above, his discrepancies mainly reflect the uncertainty in the astronomical latitudes.

It is also interesting to compare the accuracy of this triangulation with that of the French arc measurement at the Arctic circle. In that case the total distance along the triangulation network was found to be slightly more than 100 000 m. The error in this was shown through a more accurate measurement by Svanberg (1805) to be about 50 m. This makes a relative error of 1 : 2000. In the Baltic Sea triangulation via the Åland Islands the total difference in longitude along the triangulation is 3 ½ ° or nearly 200 000 m. From Table 1 we find that a scale error in this quantity hardly can exceed 5″ - 10″ or some 100 m. This makes a relative error hardly exceeding 1 : 2000. Thus the triangulation of Åland seems to have been of the same quality as the one by the French expedition in the north.

5. The Russian interest

After Finland and Åland had been ceded by Sweden to Russia in 1809, the Russians decided to build a large fortress on Åland and to move a part of their Baltic naval fleet to these islands. This might be a background for a peculiar Russian interest in the coordinates from Gadolin's triangulation across Åland. What happened was the following.

The original report of Gadolin (1757) was for unknown reasons never printed. When Hällström (1815) recomputed Gadolin's triangulation, his report was for unknown reasons not printed either. After Hällström had died, his brother sent his manuscript of the report to the Finnish Society of Sciences (in 1839). Soon after that, the Russian navy hastened to borrow the manuscript but never returned it. After some years they returned a copy of it instead, certified by two persons to be reasonably true. Since the original manuscript now had become inaccessible it was not printed this time either. Not until at the end of the century was Hällström's (1815) report printed, but now based on the Russian copy of it (see Moberg, 1893). The original manuscript probably no longer exists.

However, an earlier version of the original manuscript does in fact exist, in the Geodetic Archives of the National Land Survey of Sweden. This can be seen to be the "pre-original" manuscript, since Hällström here in a few cases has left an empty space in the text for later filling in missing details. Moreover, a complete manuscript is kept in the Archives of the National Maritime Administration of Sweden. The present author, therefore, now has compared the coordinates in the pre-original as well as in the complete manuscript with those in the Russian copy. After nearly 200 years it can be confirmed that almost all coordinates of the Russian copy are correct. There are a few erroneous planar coordinates but then the corresponding final geodetic coordinates (latitude and longitude) are correct. In one case, Getaberg, the planar coordinates are correct but the final longitude is in error by 5′; this error, however, occurs already in the complete manuscript.

The Russian interest for this triangulation may be considered as a further illustration of its accuracy: When the Russian navy got hold of the document and did not want to return it, the measurements in the document were nearly 100 years old, but obviously still considered valuable for their hydrographers.

6. Concluding remarks

We have found that the triangulation around 1750 between Finland and Sweden across the Åland Islands has a very low standard deviation in relative position, 2″ in latitude and 9″ in longitude. We also have noted that this triangulation for mapping purposes seems to be of a similar quality as the French triangulation for scientific purposes at the Arctic circle.

Altogether, the early Åland mapping triangulation appears to have a quite remarkable accuracy bearing in mind that it was a pioneering work in the middle of the 1700s. This might partly be due to the fact that the observer was able to use an instrument originally built for the use of the French Academy of Sciences at the Arctic circle.

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