Small Publications in Historical Geophysics

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Reanalysing Astronomical Coordinates of Old Fundamental Observatories using Satellite Positioning and Deflections of the Vertical

Martin Ekman & Jonas Ågren

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1. Background

In the 1700s and the 1800s each country performing national mapping established a national astronomical observatory as a fundamental point for positioning. Here the latitude and the longitude were determined astronomically with the greatest possible accuracy. These coordinates served as starting values for determinations of coordinates of other points in the country, e.g. through triangulation.

Today when positions can be easily and accurately determined through satellites (although also this method is ultimately based on stars because of the way the global reference systems are defined), these old observatories are no longer needed for positioning purposes. On the other hand, satellite positioning, together with a computation of deflections of the vertical due to irregularities in the Earth's gravity field, now will allow a redetermination of the astronomical latitude and longitude of these observatories. Thereby, the quality of the old determinations may now be investigated through an independent method. This is the main purpose of this publication.

In an earlier publication by the authors (Ekman & Ågren, 2009) such a redetermination of the latitude of Tycho Brahe's ruined observatory of Uranienborg (founded 1576) was made. In the present publication we will redetermine both the latitudes and the longitudes of some still existing fundamental observatories. Comparing our satellite-based results with the historical star-based results we will be able to find the actual errors of the old latitude and longitude determinations.

The two oldest observatory buildings in the world preserved more or less in their original state, although later expanded, are Paris (established 1667) and Greenwich in London (established 1675). One of the oldest observatory buildings kept in its original state without considerable later expansion is Stockholm (established 1748). Also the old observatory tower of København, a tower connected to a church, is still there (Rundetårn [Round tower], erected 1637), although the original observatory buildings themselves on top of the tower are long since gone. The observatory coordinates to be investigated here are those of Stockholm and København, and also those of the Greenwich observatory containing the international zero meridian.

2. Coordinates from astronomical positioning

The latitude of an observatory was determined through observing the altitude of a star. Normally the altitude was observed at the upper culmination of the star, occurring at its meridian transit in the south; preferably one used a star close to zenith, where refraction is (almost) zero. Denoting the altitude of the star by h, the latitude Φ of the observation point can be calculated as

$$\Phi = \delta + 90° - h \tag{1}$$

δ being the declination of the star.

At the Stockholm observatory (see Figure 1) the first official latitude was determined by Wargentin (1759), applying (1) to 59 star observations and then taking the average of these. His result was Φ = 59°20′31.3″. This value was used in connection with a triangulation along the coasts of Sweden, then comprising most of the coasts of the Baltic Sea. In the next century a new official latitude was determined by Cronstrand (1811). It was based partly on the old one of Wargentin recalculated by Cronstrand with a new star catalogue, and partly on a relatively small number of observations of his own. The result was Φ = 59°20′34.8″. This value was used as a basis for a renewed triangulation covering also the inland parts of Sweden. Only a few decades later Selander (1835) made a complete redetermination of the latitude, using 108 star observations (and a different method from that above). His result was Φ = 59°20′33.8″, but the earlier value continued to be used for the triangulation.

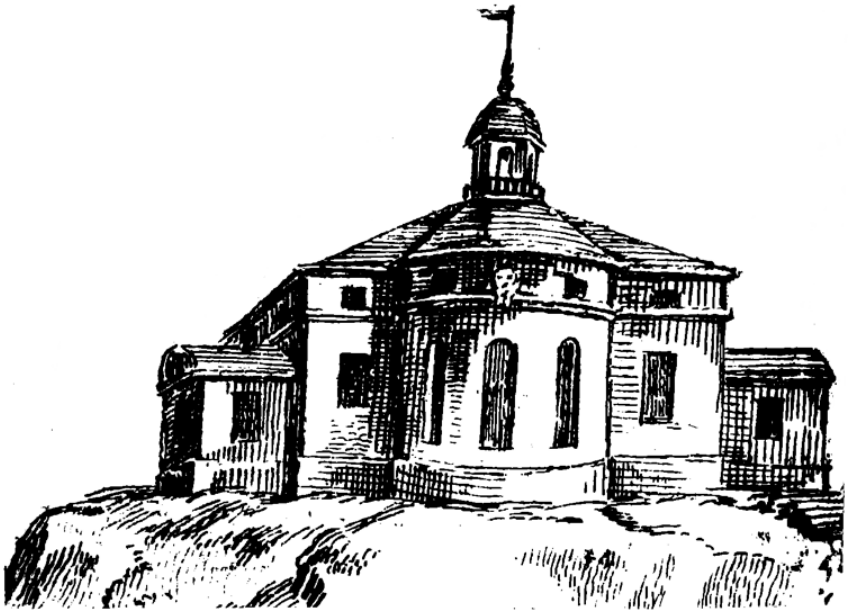

Figure 1. The old observatory of Stockholm (Wargentin, 1761)

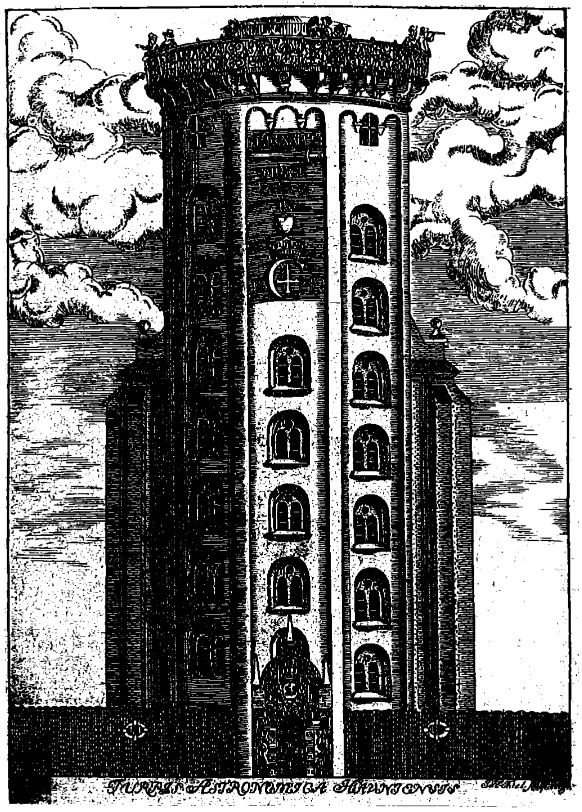

Figure 2. The old observatory of København (Horrebow, 1735)

At the København observatory (see Figure 2) the first official latitude was determined by Bugge (1779), applying (1) to 56 star observations and then taking the average of these. His result was Φ = 55°40′57.0″. This value was used in connection with a triangulation of Denmark. In the next century a new official latitude was determined by Schumacher (1827), applying (1) to no less than 279 star observations. His result was Φ = 55°40′52.6″. This value was used as a basis for a renewed triangulation of Denmark. Later this value was adopted as a starting value also for the Swedish triangulations in the 1900s.

The longitude of an observatory was in principle determined from the difference in local time between the observatory and the Greenwich observatory on the zero meridian. The two local times T and T_0 were determined from meridian transits of stars. Then the longitude becomes

$$\Lambda = T - T_0 \qquad (2)$$

The tricky problem here was to find the two local times in (2) at one and the same instant. Originally this was achieved by observing occultations of the moons of Jupiter simultaneously at the two observatories (often with Paris as the zero meridian). Later the ship chronometer, a portable clock, allowed transporting Greenwich time across the sea and thereby made possible a more accurate determination of longitudes relative to the Greenwich observatory.

In the middle of the 1800s the largest longitude chronometer expedition in the world was performed across the North and Baltic Seas. The western part of this longitude expedition, between Greenwich and the Altona observatory (close to Hamburg), involved 42 chronometers and 16 voyages (8 + 8) forth and back; it was performed by Struve & Struve (1846). The eastern part, between Altona and the Pulkovo observatory (close to St. Petersburg), involved no less than 86 chronometers and 16 voyages; it was performed by Struve (1844). Later on further longitude connections from these observatories to the observatories of Stockholm and København were performed using the newly invented telegraph for transferring time from one observatory to another; see Fuss & Nyrén (1871) for Stockholm and Peters (1884) for København.

In this way Struve (1844) combined with Fuss & Nyrén (1871) resulted in a longitude of Stockholm of Λ = 18°03′29.8″. This value seems to have been adopted as the official longitude of the Stockholm observatory relative to the zero meridian of Greenwich. In a similar way Struve & Struve (1846) combined with Peters (1884) resulted in a longitude of København of Λ = 12°34′39.6″.

In summary, from the old astronomical determinations we have the latitude and longitude of the Stockholm observatory as

Φ = 59°20′31.3″	Wargentin (1759)
Φ = 59°20′34.8″	Cronstrand (1811)
Φ = 59°20′33.8″	Selander (1835)
Λ = 18°03′29.8″	Struve (1844), Fuss & Nyrén (1871)

In the same way we have the latitude and longitude of the København observatory,

Φ = 55°40′57.0″	Bugge (1779)
Φ = 55°40′52.6″	Schumacher (1827)
Λ = 12°34′39.6″	Struve & Struve (1846), Peters (1884)

3. Coordinates from satellite positioning

In several cases geocentric coordinates of first order triangulation stations have been determined using modern satellite positioning (GPS). Such determinations have usually been performed within national campaigns for establishing GPS coordinates on old triangulation stations. The resultant coordinates are obtained in a reference system closely related to the global systems ITRF 89 and WGS 84, the European system known as ETRS 89 (ETRF 89, EUREF 89). For the Swedish version of this (SWEREF 99) see Jivall & Lidberg (2001), for the Danish version see Jensen & Madsen (1998). For our purposes all the mentioned systems can be considered more or less identical. These geocentric coordinates can then be transferred into latitude, longitude and height relative to the Earth ellipsoid (GRS 1980).

Now, the ETRS 89 coordinates of the Stockholm observatory can be found by starting from the first order triangulation station of Kastellholmen in central Stockholm, where GPS observations have been performed. From this station the coordinate differences to the nearby station of Katarina kyrka are known from a recent triangulation of Sweden. And from that station the coordinate differences to the Stockholm observatory are known from an earlier triangulation; see Selander (1866). Putting all these together we find ETRS 89 coordinates according to the following, with latitudes and latitude differences in the left column and longitudes and longitude differences in the right column:

Kastellholmen	59 19 20.398	18 05 27.244
	- 18.976	- 45.981
Katarina kyrka	59 19 01.422	18 04 41.263
	+ 1 27.740	- 1 24.500
Stockholm observatory	59 20 29.162	18 03 16.763

Thus the latitude φ and the longitude λ of the Stockholm observatory in the ETRS 89 system are

$$\varphi = 59°20'29.16''$$
$$\lambda = 18°03'16.76''$$

These coordinates, like the astronomical ones, refer to the meridian transit instrument in the western room, to where the Stockholm meridian was moved from the eastern room in 1830.

The ETRS 89 coordinates for the København observatory can be found in a similar way by starting from the old first order triangulation station Nikolai tårn. Its coordinates are known in the old Danish system S 34, from which they can be accurately transformed into ETRS 89. From Nikolai tårn the coordinate differences to the København observatory are known through triangulation; see Peters (1884). Putting these things together we find ETRS 89 coordinates according to the following, again with latitudes and latitude difference in the left column and longitudes and longitude difference in the right column:

Nikolai tårn	55 40 43.001	12 34 52.695
	+ 10.061	- 19.905
København observatory	55 40 53.062	12 34 32.790

Thus the latitude and the longitude of the København observatory (Rundetårn) in the ETRS 89 system are

$\varphi = 55°40'53.06''$
$\lambda = 12°34'32.79''$

The satellite-derived latitudes and longitudes found above are not directly comparable with the star-derived latitudes and longitudes of Section 2. The difference is due to the deflections of the vertical which we now need to find.

4. Gravimetric deflections of the vertical

Determining a latitude by astronomical positioning means measuring vertical angles towards a star. When putting up the instrument for measuring angles it is adjusted with a spirit level. The spirit level "feels" the direction of the plumb line, or the vertical. The vertical, being the normal to the geoid, deviates from the normal to the ellipsoid. This deviation, known as the deflection of the vertical, directly affects the astronomically determined latitude.

Determining a latitude by satellite positioning means measuring distances through timekeeping of radio waves emitted from the satellites. This procedure is independent of any spirit level and, hence, does not depend on the direction of the vertical. Thus the latitude so determined is unaffected by the deflection of the vertical. The same arguments go for longitudes.

Denoting as above the star-derived or astronomical latitude and longitude by Φ and Λ, and the satellite-derived or geodetic latitude and longitude by φ and λ, we may write

$$\Phi = \varphi + \xi \tag{3}$$

$$\Lambda = \lambda + \eta / \cos \varphi \tag{4}$$

Here ξ and η are deflections of the vertical in the south-north and west-east directions, respectively.

Now, the deflection of the vertical at a certain point is nothing but the inclination of the geoid relative to the ellipsoid at that point. Thus the deflection of the vertical ξ can be computed as the derivative of the geoid height N in the south-north direction, and the deflection of the vertical η as the derivative of the geoid height N in the west-east direction,

$$\xi = -\frac{\partial N}{R \, \partial\varphi} \tag{5}$$

$$\eta = -\frac{\partial N}{R \cos \varphi \, \partial\lambda} \tag{6}$$

R being the mean radius of the Earth.

The geoid and, thereby, the deflections of the vertical are due to the irregular mass distribution within the Earth. Hence the geoid can be computed from a detailed and global knowledge of the Earth's gravity field. Such a knowledge has only been achieved during the last decades. Modern geoid computations are based on a combination of satellite orbit perturbations, surface gravity anomalies, and digital terrain models. The most recent global geoid model is EGM 2008 of Pavlis et al (2008). This is given as a spherical harmonic series expansion up to degree and order 2160, corresponding to a minimum resolution (half wave-length) of 0.08°. We also have the recent geoid model SWEN08_RH2000 over Sweden and some adjacent areas by Ågren (2009), based on KTH08 by Ågren et al (2009). This is computed as a grid with density 0.02°. Over land areas this regional model can be considered slightly more accurate than the global one; it is illustrated as a geoid height map in Figure 3.

According to these geoid models we obtain the following deflections of the vertical at the Stockholm observatory:

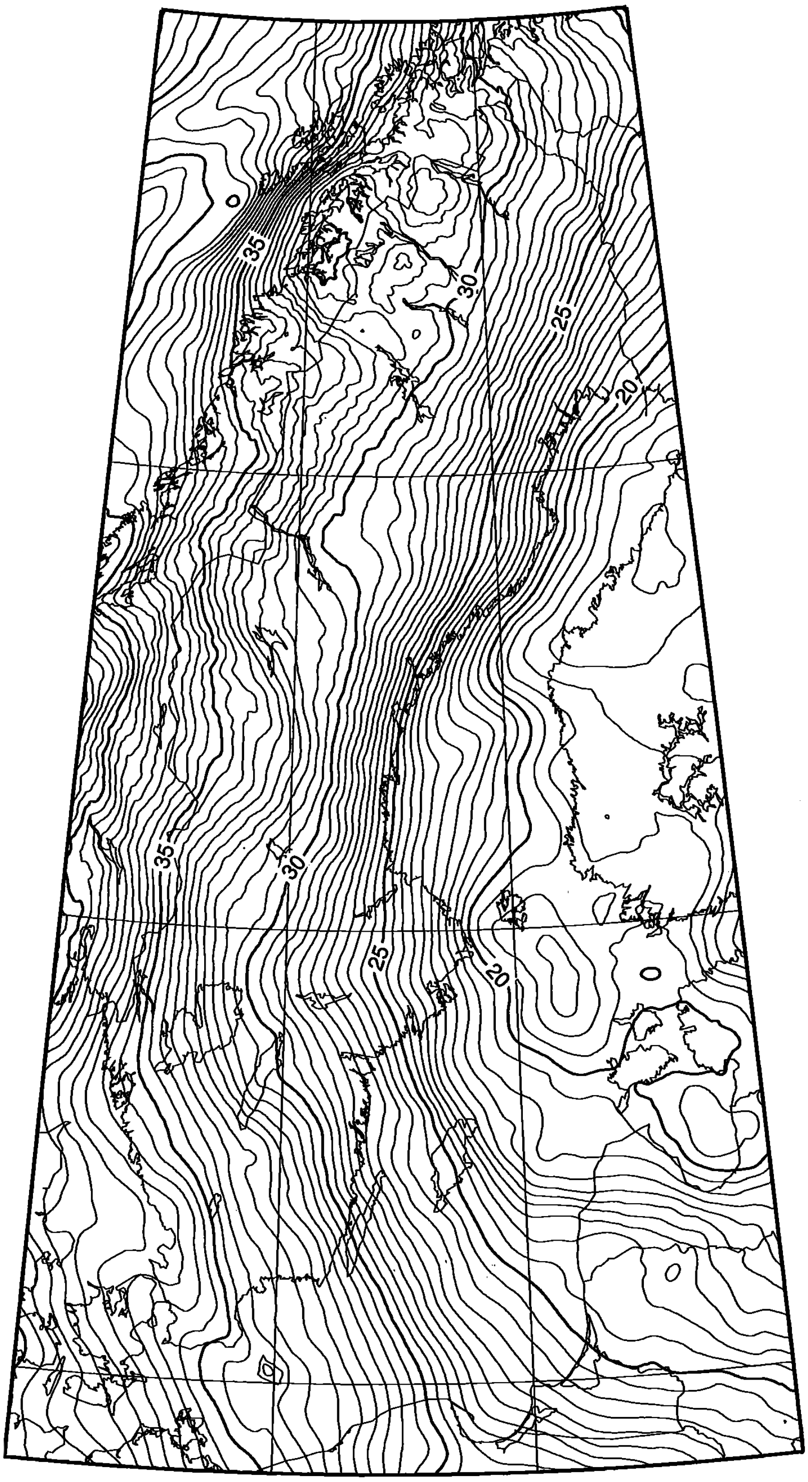

Figure 3. Map of geoid heights (m) over Sweden and adjacent areas (Ågren, 2009).

Regional model	Global model
$\xi = 3.89''$	$\xi = 4.10''$
$\eta / \cos\varphi = 12.69''$	$\eta / \cos\varphi = 11.87''$

Correspondingly we obtain the following deflections of the vertical at the København observatory:

Regional model	Global model
$\xi = 0.27''$	$\xi = 0.07''$
$\eta / \cos\varphi = 5.94''$	$\eta / \cos\varphi = 5.76''$

The models differ by some 0.2″; this agrees with the estimated uncertainty of the deflections. An exception is the west-east deflection at Stockholm; see further the end of Section 5.

With the deflections of the vertical known we now are in the position to calculate the astronomical latitudes and longitudes of Stockholm and København. In doing so we will use the regional model which, according to our judgement, in these cases is slightly better.

5. Final results: Stockholm and København

For the observatories of Stockholm and København we have found the satellite-derived coordinates in Section 3 and the gravimetric deflections of the vertical in Section 4. Applying (3) and (4), using the regional geoid model, we obtain the astronomical coordinates for the Stockholm observatory:

$\Phi = 59^\circ 20' 33.05''$
$\Lambda = 18^\circ 03' 29.45''$

Correspondingly we obtain for the København observatory:

$\Phi = 55^\circ 40' 53.33''$
$\Lambda = 12^\circ 34' 38.73''$

The satellite-derived coordinates used for the calculations can be considered error-free. As the uncertainty in the deflections of the vertical according to above can be estimated at 0.2″, the same uncertainty will be valid for the astronomical coordinates obtained.

The astronomical coordinates thus found can now be compared with the old ones determined astronomically through star observations in Section 2. These comparisons are summarized below, in Tables 1 and 2. In Table 1 the astronomical coordinates for the Stockholm observatory are given, in the first column our own values from satellite positioning and gravimetry, in the second column the old values from star observations. The third column gives the difference between our values and the old ones. In Table 2 the same quantities for the København observatory are given.

Table 1. Astronomical coordinates for the Stockholm observatory: our values from satellites and gravimetry, the old values from stars, and the differences between the two.

	Sat + grav	Stars		Difference
Lat.	59°20′33.0″	59°20′31.3″	(Wargentin, 1759)	- 1.7
		59°20′34.8″	(Cronstrand, 1811)	+ 1.8
		59°20′33.8″	(Selander, 1835)	+ 0.8
Long.	18°03′29.4″	18°03′29.8″	(Struve, 1844 a.o.)	+ 0.4

Table 2. Astronomical coordinates for the København observatory: our values from satellites and gravimetry, the old values from stars, and the differences between the two.

	Sat + grav	Stars		Difference
Lat.	55°40′53.3″	55°40′57.0″	(Bugge, 1779)	+ 3.7
		55°40′52.6″	(Schumacher, 1827)	- 0.7
Long.	12°34′38.7″	12°34′39.6″	(Struve, 1846 a.o.)	+ 0.9

Since the uncertainty in our coordinates in the first column is only about 0.2″ as stated above, the differences in the third column may be interpreted as the actual errors in the old star-derived coordinates, given in the second column. This leads us to some informative conclusions.

Starting with the latitudes we find that Wargentin's (1759) latitude determination of the Stockholm observatory was an excellent one for his time, showing an error of only 1.7". Cronstrand's (1811) attempt to improve Wargentin's result half a century later must be characterized as unsuccessful. On the other hand, Selander (1835) succeeded in reducing the error to 0.8", one half of that of Wargentin. At the København observatory Bugge's (1779) latitude determination shows an error of 3.7", twice that at the Stockholm observatory. Half a century later Schumacher (1827) succeeded in reducing the error at København to 0.7", one fifth of Bugge's error, and one half of that at Stockholm at that time.

Turning to the longitudes we first note that their errors should be almost wholly due to the ship chronometer expeditions from Greenwich described in Section 2, the additional telegraph connections being in comparison more or less error-free. Thus the longitude results obtained in Tables 1 and 2 yield an interesting possibility to evaluate the quality of the largest longitude chronometer expeditions in the world, those of Struve in the 1840s.

For København (Table 2), telegraphically connected to Altona, we find an error in longitude of 0.9". This can be interpreted as the actual error in Struve's (1846) longitude expedition across the North Sea between the Greenwich and Altona observatories. For Stockholm (Table 1), telegraphically connected to Pulkovo, we find an error in longitude of 0.4". The interpretation of this is an actual error of 0.4 - 0.9 = - 0.5" in Struve's (1844) longitude expedition across the Baltic Sea between the Altona and Pulkovo observatories. The longitude errors of 0.9" and - 0.5" correspond to errors in time of only some 0.06 s and - 0.03 s, respectively. (In 1851 the Greenwich meridian was moved 0.3" to the east (cf. Section 6), but this seems to have been ignored in the official longitudes of the other observatories. Correcting for this the longitude errors would become 0.6" and - 0.5", corresponding to 0.04 s and - 0.03 s in time.) This must be considered a remarkable achievement bearing in mind that the astronomically determined time at the Greenwich observatory was transported by portable ship clocks across the sea to the other observatories. It could be mentioned here that the first longitude expedition of this kind was performed two decades earlier between Greenwich and Altona by Schumacher (1827a) in cooperation with the Greenwich observatory. That result shows an error of 7.4" or 0.5 s, so Struve's result was an improvement by one order of magnitude.

In addition to the old determinations above there is also a "modern" astronomical latitude determination of the Stockholm observatory, by Rosén (1879), and a "modern" astronomical longitude difference determination between the Stockholm and København observatories, by Lindhagen et al

(1890). These determinations have uncertainties comparable to (or smaller than) those of our own values. This gives a possibility to check the agreement between the astronomical determinations on one hand, and the satellite determinations and gravimetric deflections of the vertical on the other hand; see Table 3. The agreement turns out to be excellent. The discrepancies amount to 0.1″, a figure that could be expected from astronomical and gravimetric considerations. However, this agreement, using the regional geoid model, also reveals that the global geoid model in Section 4 must have a local error in the west-east deflection at Stockholm.

Table 3. Some astronomical coordinates for the Stockholm and København observatories: our values from satellites and gravimetry, "modern" values from stars, and the differences between the two.

	Sat + grav	Stars		Difference
Lat. St.	59°20′33.0″	59°20′32.9″	(Rosén, 1879)	- 0.1
Long. diff. St - Kø	5°28′50.7″	5°28′50.8″	(Lindhagen et al, 1890)	+ 0.1

6. Additional results: Greenwich

In addition we have made a special investigation of the Greenwich observatory with the international zero meridian. Its ETRS 89 coordinates are known: φ = 51°28′40.12″, λ = - 00°00′05.31″. These coordinates refer to the meridian transit instrument in the Airy room, to where the Greenwich meridian was moved from the Bradley room in 1851. Calculating the deflections of the vertical for this position, applying (5) and (6) to the global geoid model, we find ξ = - 2.15″, $\eta/\cos\varphi$ = 5.51″. Adding these quantities according to (3) and (4) we obtain the astronomical coordinates of the Greenwich observatory given in Table 4. This table also gives the discrepancy between the obtained astronomical longitude and the astronomical zero longitude definition, ignoring here the small effects of polar drift and continental drift. The discrepancy turns out to be 0.2″. This is in good agreement with the estimated uncertainty of the deflections of the vertical.

Table 4. Astronomical coordinates for the Greenwich observatory (international zero meridian).

	Sat + grav	Stars	Discrepancy
Lat.	51°28′38.0″	-	-
Long.	00°00′00.2″	00°00′00.0″ (Definition)	0.2

7. Conclusions

We have calculated the astronomical latitudes and longitudes of the old fundamental observatories of Stockholm, København and Greenwich by combining satellite positioning and gravimetric deflections of the vertical. These coordinates have then been compared with old ones determined from star observations. The results are summarized in Tables 1 and 2, with some additional results in Tables 3 and 4. The main conclusions from these tables are:

1. The errors in latitude reached below 1″ with the determinations of Schumacher (1827) and Selander (1835).

2. The errors in longitude reached below 1″ with the sea chronometer expeditions of Struve (1844, 1846), later extended by telegraph connections.

3. The agreement between "modern" star-derived coordinates on one hand and present-day satellite-derived coordinates combined with deflections of the vertical on the other hand is within some 0.2″.

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