Small Publications in Historical Geophysics

No. 23

A Study of Celsius' Astronomical Latitude Determination of the Uppsala Observatory using Satellite Positioning and Deflections of the Vertical

Martin Ekman & Jonas Ågren

Summer Institute for Historical Geophysics
Åland Islands

2012

Small Publications in Historical Geophysics

No. 23

A Study of Celsius' Astronomical Latitude Determination of the Uppsala Observatory using Satellite Positioning and Deflections of the Vertical

Martin Ekman & Jonas Ågren

Contents

Summer Institute for Historical Geophysics
Åland Islands

2012

1. The observatory of Uppsala

In 1739 Anders Celsius, today mostly known for his temperature scale, founded the observatory of Uppsala; see Figure 1. It was intended to be not only the astronomical observatory of the University of Uppsala, but also a national observatory for astronomical positioning and mapping. The latter function was after some 20 years taken over by the observatory of Stockholm, then newly founded by the Royal Swedish Academy of Sciences. However, in the middle of the 1700s the Uppsala observatory was a centre for latitude and longitude determination, and also for the study of quantities like gravity, magnetism, temperature and air pressure.

A fundamental task for Celsius at his new observatory was to determine its latitude with the greatest possible accuracy. This work was based on the observations of stars. Celsius thereby developed and applied a method of his own to eliminate disturbing effects.

Today satellite positioning, together with a computation of deflections of the vertical due to irregularities in the Earth's gravity field, will allow a redetermination of the astronomical latitude of the Uppsala observatory. Thereby, the quality of Celsius' determination may now be investigated. This is the main purpose of this publication.

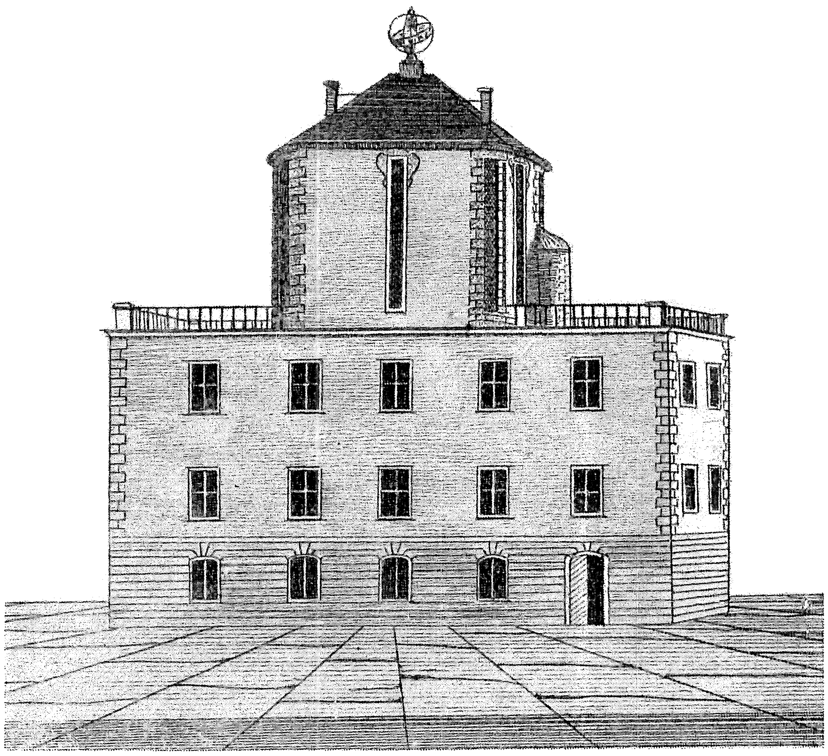

Figure 1. The observatory of Uppsala at the time of Celsius. (Engraving by F Akrel 1769.)

In an earlier publication by the authors (Ekman & Ågren, 2009) such a redetermination of the latitude of Tycho Brahe's observatory of Uranienborg was made. In a more recent publication by the same authors (Ekman & Ågren, 2010) similar redeterminations of latitude and longitude of the fundamental observatories of Stockholm and København were performed. Now the turn has come to Celsius' observatory of Uppsala. This was not included earlier because Celsius never published a latitude of his observatory before he died quite unexpectedly. However, from Celsius hand-written notes he left at his death (now in the University Library of Uppsala) it is possible to find a latitude result. Comparing our satellite-based result with the star-based result of Celsius we will be able to find the actual error in Celsius' latitude determination. We will also compare Celsius' error for the Uppsala observatory with the contemporary one for the København observatory, and also with somewhat later ones for both the Stockholm and København observatories. This will enable us to judge the quality of Celsius' method.

Celsius introduced the Uppsala observatory as a zero meridian for the Nordic countries; thus the longitude of the observatory was by definition zero. The longitude of Uppsala relative to the more international zero meridian in Paris could at that time not be determined with the same accuracy as the latitude and is therefore not investigated here; for a discussion of this see Ekman (2011).

2. Celsius' latitude determination

Celsius determined the latitude of the Uppsala observatory through an interesting combination of the two basic principles for latitude determination. The first principle is based on observing circumpolar stars. Such a star will transit the meridian twice a day, once on the upper side of the pole, upper culmination, and once on the lower side, lower culmination. We denote the altitude (height) of the star at upper culmination by h_u and at lower culmination by h_l. Then the altitude of the pole can be identified as the mean value of the two observed altitudes of the star, and this is equal to the latitude Φ of the observation point:

$$\Phi = \frac{h_u + h_l}{2} \tag{1}$$

The second principle is based on observing the altitude of a star only at upper culmination. In that case also the declination of the star is needed. Denoting the altitude of the star by h, the latitude Φ of the observation point can be calculated as

$$\Phi = \delta + 90° - h \quad (2)$$

δ being the declination of the star.

When measuring the above altitudes of stars, a disturbing error source is the refraction of the star light in the atmosphere. This is tricky to handle in (1), requiring an uncertain correction calculation. In (2) it might to a large extent be eliminated by choosing a star close to zenith where refraction is close to zero. On the other hand, another disturbing error source is introduced in (2) by the declinations of stars being not sufficiently well known. This error source does not appear in (1). Celsius (1739) developed and soon applied a method to both circumvent the refraction problem and reduce the declination problem, by combining the two formulae above in the following way.

First, the latitude is determined through (1), using a circumpolar star with the upper culmination close to zenith. For Uppsala, with $\Phi \approx 60°$, the altitudes then become $h_u \approx 90°$ and $h_l \approx 30°$. This makes refraction close to zero for the upper culmination, but considerable for the lower culmination. The obtained latitude will be in error by half of the refraction for the lower culmination.

Second, in (2) use another star which is not circumpolar but has its upper culmination at (almost) the same altitude as the lower culmination of the circumpolar star. Thus here $h \approx h_l \approx 30°$, yielding (almost) the same refraction, and $\delta \approx 0°$. Now, put into (2) the latitude obtained from (1). The discrepancy between 90° and the sum of the other quantities in (2) is then 1 ½ times the refraction at the lower culmination in (1).

Third, subtract 1/3 of this discrepancy from the latitude obtained by (1). The result should be the true latitude, without refraction. Moreover, the uncertainty in the declination entering into the latitude from (2) is reduced to one third.

We may summarize Celsius' method, described above according to his own way of using it, in the following formula for computing the latitude:

$$\Phi = \frac{h_u + h_l}{2} - \frac{1}{3}\left[\frac{h_u + h_l}{2} + h - \delta - 90°\right] \quad (3a)$$

where $h \approx h_l$. Apparently, then, h and h_l cancel each other in (3a), and consequently also the refraction affecting these two quantities. As can be seen also the declination and thereby its uncertainty is reduced to one third.

Let us now denote the latitude determined from (1) only by Φ_1 and the corresponding latitude determined from (2) only by Φ_2. Inserting (1) and (2) into (3a) we can reformulate (3a) to give it a new look,

$$\Phi = 2/3\ \Phi_1 + 1/3\ \Phi_2 \qquad (3b)$$

Thus Celsius' favourable method for determining latitude is equivalent to taking a weighted mean of (1) and (2), whereby (1) is given twice the weight of (2).

In Celsius' hand-written notes there are a number of latitude determinations made now and then during the years 1739 – 1741, using the above method. The latitudes obtained by him vary between 59°51′35″ and 59°51′42″. The observations were made in a provisional observatory while the official one was under construction. This provisional observatory seems to have been situated some 30 – 40 m to the north. (Further comments on this are given in Section 5.) Hence the latitudes obtained should be reduced by 1″ to refer to the official observatory. Doing so and taking the average of his results we find the latitude

$$\Phi = 59°51'39''$$

We consider this the final result of Celsius' latitude determinations of the Uppsala observatory. In a hand-written table by Celsius' assistant Hiorter summarizing astronomical latitude determinations in eastern Sweden as a basis for producing a map, the latitude of Uppsala is also given by this value (after the same reduction of 1″). It is not always quite clear from Celsius' notes whether a calculation by him is actually a latitude determination or some other calculation involving the latitude; hence no decimal is added to the seconds here.

3. The latitude from satellite positioning

We now switch to modern times. In several cases geocentric coordinates of first order triangulation stations have been determined using modern satellite positioning (GPS). Such determinations have usually been performed within national campaigns for establishing GPS coordinates on old triangulation stations. The resultant coordinates are obtained in a reference system closely related to the global systems ITRF 89 and WGS 84, the European system known as ETRS 89, or rather its Swedish version SWEREF 99; see Jivall & Lidberg (2000). For our purposes all the mentioned systems can be considered more or less identical. These geocentric coordinates are then transferred into latitude, longitude and height relative to the Earth ellipsoid (GRS 1980). Because of such coordinates being determined on old triangulation stations, it is possible to perform accurate transformations between old reference systems connected to the triangulations and the modern satellite-based reference systems.

Now, the satellite-based ETRS 89 coordinates of the Uppsala observatory can be found by starting from the first order triangulation station of Uppsala cathedral, more specifically its northern tower. Its coordinates are known in the Swedish system RT 90 connected to the last national triangulation. These coordinates can be accurately transformed into ETRS 89. From local measurements made, the coordinate differences to the southern tower are established. From this southern tower of the Uppsala cathedral the coordinate differences to the Uppsala observatory are known through an earlier triangulation; see Selander (1866). Putting these things together we find ETRS 89 coordinates according to the following, with latitudes and their differences in the left column and longitudes and their differences in the right column.

Uppsala cathedral, N tower	59 51 29.63	17 37 57.78
	- 0.71	- 0.02
Uppsala cathedral, S tower	59 51 28.92	17 37 57.76
	+ 6.26	+ 15.27
Uppsala observatory	59 51 35.18	17 38 13.03

Thus the latitude φ and the longitude λ of the Uppsala observatory (Celsius' observatory) in the ETRS 89 system are

$$\varphi = 59°51'35.18''$$
$$\lambda = 17°38'13.03''$$

These coordinates, like the astronomical ones, refer to the centre of the observatory. The longitude is included here for the sake of completeness, although not used in the end for reasons explained in Section 1.

The satellite-derived latitudes and longitudes found above are not directly comparable with the star-derived latitude of Section 2. The difference is due to the deflections of the vertical which we now need to find, thereby adopting the same method as in Ekman & Ågren (2010), from where much of the following section is taken.

4. The gravimetric deflection of the vertical

Determining a latitude by astronomical positioning means measuring vertical angles towards a star. When putting up the instrument for measuring angles it is adjusted with a spirit level. The spirit level "feels" the direction of the plumb line, or the vertical. The vertical, being the normal to the geoid, deviates from the normal to the ellipsoid. This deviation, known as the deflection of the vertical, directly affects the astronomically determined latitude.

Determining a latitude by satellite positioning means measuring distances through timekeeping of radio waves emitted from the satellites. This procedure is independent of any spirit level and, hence, does not depend on the direction of the vertical. Thus the latitude so determined is unaffected by the deflection of the vertical. The same arguments go for longitudes, which are included here for a wider understanding of the nature of the deflections in the Uppsala area.

Denoting the star-derived or astronomical latitude and longitude by Φ and Λ, and the satellite-derived or geodetic latitude and longitude by φ and λ, we may write

$$\Phi = \varphi + \xi \qquad (4)$$

$$\Lambda = \lambda + \eta / \cos \varphi \qquad (5)$$

Here ξ and η are deflections of the vertical in the south-north and west-east directions, respectively.

Now, the deflection of the vertical at a certain point is nothing but the inclination of the geoid relative to the ellipsoid at that point. Thus the deflection of the vertical ξ can be computed as the derivative of the geoid height N in the south-north direction, and the deflection of the vertical η as the derivative of the geoid height N in the west-east direction,

$$\xi = -\frac{\partial N}{R \, \partial\varphi} \qquad (6)$$

$$\eta = -\frac{\partial N}{R \cos\varphi \, \partial\lambda} \qquad (7)$$

R being the mean radius of the Earth.

The geoid and, thereby, the deflections of the vertical are due to the irregular mass distribution within the Earth. Hence the geoid can be computed from a detailed and global knowledge of the Earth's gravity field. Such a knowledge has only been achieved during the last decades. Modern geoid computations are based on a combination of satellite orbit perturbations, surface gravity anomalies, and digital terrain models. The most recent global geoid model is EGM 2008 of Pavlis et al (2008). This is given as a spherical harmonic series expansion up to degree and order 2160, corresponding to a minimum resolution (half wave-length) of 0.08°. We also have the recent geoid model SWEN08_RH2000 over Sweden and some adjacent areas by Ågren (2009), based on KTH08 by Ågren et al (2009). This is computed as a grid with density 0.02°. Over land areas this regional model can be considered slightly more accurate than the global one; it is illustrated as a geoid height map in Figure 2.

According to these geoid models we obtain the following deflections of the vertical at the Uppsala observatory:

Regional model	Global model
ξ = 0.89″	ξ = 0.96″
$\eta/\cos\varphi$ = 15.91″	$\eta/\cos\varphi$ = 14.76″

The models differ by 0.1″ in latitude; this is within the estimated uncertainty of 0.2″ of the deflections. In longitude the difference between the models is too large relative to the mentioned uncertainty. Interestingly enough the same thing was found for Stockholm by Ekman & Ågren (2010). In that case we could show that the regional model was correct while the global model was erroneous. We might suspect the same thing here.

We note that the deflection of the vertical is very small in latitude, while it is quite large in longitude. This means that the geoid in the Uppsala area has a considerable inclination in the west-east direction but almost none in the south-north direction. This is a quite general pattern over much of Sweden, as shown by Figure 2.

With the deflection of the vertical known we now are in the position to calculate the astronomical latitude of the Uppsala observatory. In doing so we will use the regional model which, according to our judgement, in this case is slightly better.

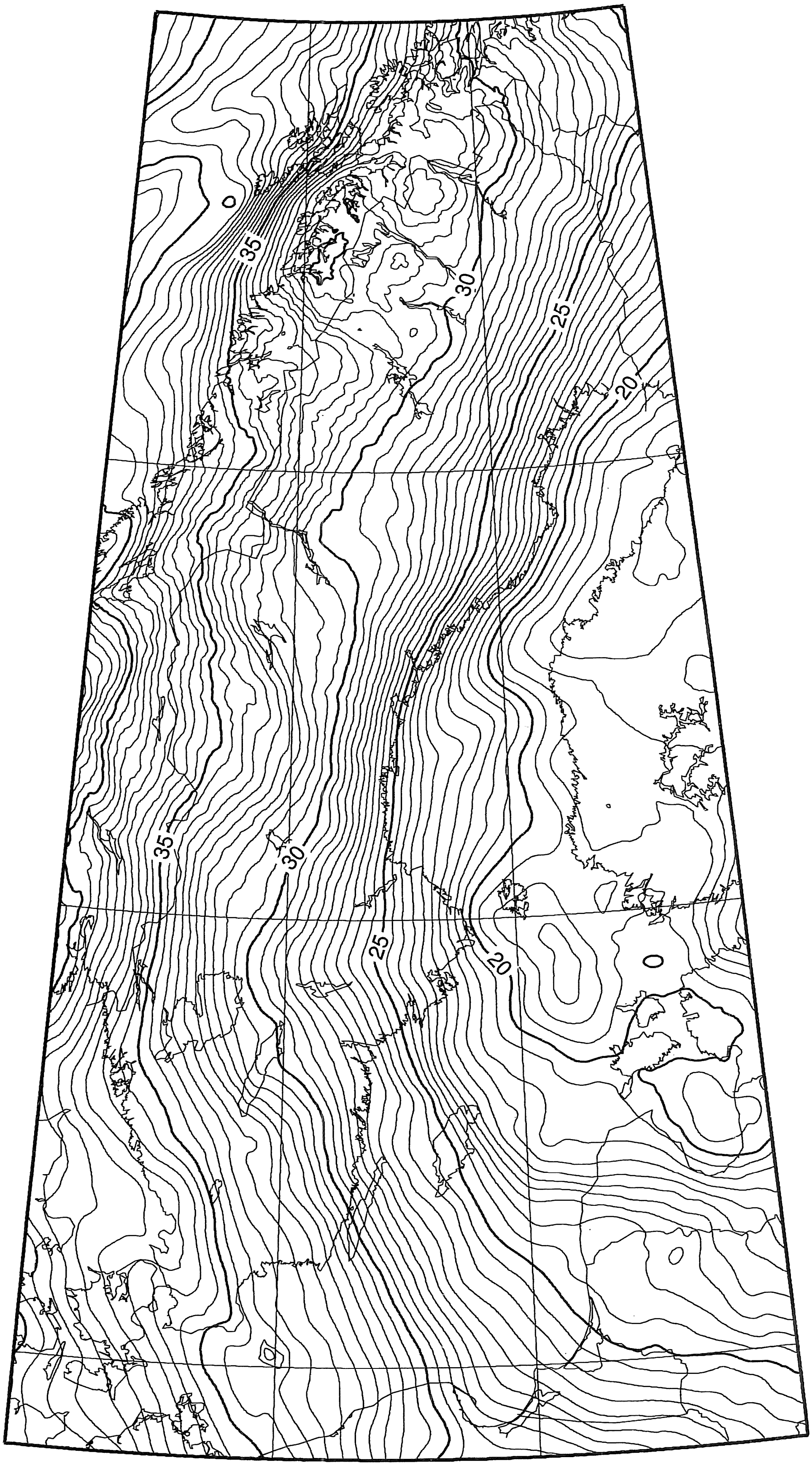

Figure 2. Map of geoid heights (m) over Sweden and adjacent areas (Ågren, 2009).

5. Final results

For the observatory of Uppsala we have found the satellite-derived coordinates in Section 3 and the gravimetric deflections of the vertical in Section 4. Applying (4) and (5), using the regional geoid model, we obtain the astronomical latitude and longitude for the Uppsala observatory (Celsius' observatory):

Φ = 59°51′36.1″
Λ = 17°38′28.9″

The satellite-derived coordinates used for the calculations can be considered error-free. As the uncertainty in the deflections of the vertical according to above can be estimated at 0.2″, the same uncertainty will be valid for the astronomical coordinates obtained.

The astronomical latitude thus found can now be compared with the old one determined through Celsius' star observations in Section 2. This comparison may be summarized as follows:

Our latitude	Celsius' latitude	Difference
59°51′36.1″	59°51′39″	3″

Since the uncertainty in our latitude is only about 0.2″ as stated above, the difference may be interpreted as the absolute error in Celsius' latitude. Thus we conclude that Celsius' latitude is in error by 3″ only.

Let us now compare Celsius' latitude error at Uppsala with contemporary and later latitude errors at the København and Stockholm observatories in the 1700s. These results, referring to "official" latitude determinations, are given below; they are based on Ekman & Ågren (2010) and Ekman (2011):

	Observatory	Original reference	Latitude error
Contemporary	København	Horrebow (1732)	+ 6″
	Uppsala	Celsius (1739)	+ 3″
Later	Stockholm	Wargentin (1759)	- 2″
	København	Bugge (1779)	+ 4″

We conclude that Celsius' latitude determination of the Uppsala observatory is more successful than the contemporary one of the København observatory. It should be noted here that Horrebow at København is known for a method of his own to eliminate refraction (but not reduce the declination problem, including the then unknown nutation). Furthermore, Celsius' determination is of the same quality as the ones performed one generation later at København and Stockholm (the latter by one of Celsius' former assistants), in spite of the fact that by that time improved declinations were available.

Finally a small remark: In Section 2 we noted that Celsius' observations were made in a provisional observatory, for which a reduction to the official observatory by 1″ in latitude was made. The exact location of his provisional observatory is not known with absolute certainty; there is a small possibility that is was one or a few seconds further north. However, this would only bring his latitude error closer to zero, so the error found here is in any case not too optimistic.

6. Conclusions

We have calculated the astronomical latitude of the old fundamental observatory of Uppsala by combining satellite positioning and the gravimetric deflection of the vertical. This latitude has then been compared with the old one determined by Celsius from star observations, whereby he used a method of his own to deal with refraction and declination problems. We find that Celsius' latitude result is in error by only 3″. This is clearly better than the contemporary one at the København observatory, although also there a special method to handle refraction was used. Moreover, Celsius' result can compete with the ones performed one generation later at København and Stockholm, although by that time improved declinations were available.

On the whole, Celsius' method of determining the latitude must be considered very successful for its time.

References

Ågren, J (2009): Beskrivning av de nationella geoidmodellerna SWEN08_RH2000 och SWEN08_RH70. Lantmäteriet, Rapporter i Geodesi och Geografiska informationssystem, 2009:1, 33 pp.

Ågren, J, Sjöberg, L E, Kiamehr, R, (2009): The new gravimetric quasigeoid model KTH08 over Sweden. Journal of Applied Geodesy, 3, 143-153.

Bugge, T (1779): Beskrivelse over den Opmaalings Maade some er brugt ved de danske geographiske Karter, med tilføiet trigonometrisk Karte over Siæland, og de der henhørende Triangler, beregnede Longituder og Latituder, samt astronomiske Observationer. København, 132 pp.

Celsius, A (1739): Elevationem veram poli octante circuli non minorem, sine cognitione refractionis siderum, invenire. Acta literaria et scientiarum Sveciae, 1739, 453-454.

Ekman, M (2011): Where on Earth are we? Using the sky for mapping the Nordic countries 1500 – 2000. Summer Institute for Historical Geophysics, Åland Islands, 138 pp.

Ekman, M, & Ågren, J (2009): A study of Tycho Brahe's astronomical latitude determination of Uranienborg using satellite positioning and deflections of the vertical. Small Publications in Historical Geophysics, 18, 13 pp.

Ekman, M, & Ågren J (2010): Reanalysing astronomical coordinates of old fundamental observatories using satellite positioning and deflections of the vertical. Small Publications in Historical Geophysics, 21, 18 pp.

Horrebow, P (1732): Atrium astronomie sive de inveniendis refractionibus, obliqvitate eclipticae, atqve elevatione poli tractatus. København, 96 pp.

Jivall, L, & Lidberg, M (2000): SWEREF 99 – An updated EUREF realisation for Sweden. International Association of Geodesy, Subcommission for the European Reference Frame, 9, 167-175.

Pavlis, N K, Holmes, S A, Kenyon, S C, Factor, J K (2008): An Earth Gravitational Model to Degree 2160: EGM 2008. European Geosciences Union, General Assembly, 2008.

Selander, N H (1866): Ortbestämmelser i Sverige verkställde af Topografiska Corpsen åren 1814 – 1849. Stockholm, 54 pp. (Author anonymous.)

Wargentin, P (1759): Stockholms observatorii pol-högd bestämd. Kongl. Vetenskaps Academiens Handlingar, 20, 305-314.

Archived data

Celsius' latitude determinations at the Uppsala observatory 1739 – 1741, University Library of Uppsala.

Acknowledgement

We would like to thank Britt Marie Ekman at the Geodetic Archives at the National Land Survey of Sweden for supplying relevant coordinates of the towers of the Uppsala cathedral, and Eric Stempels at the Astronomy Division at the University of Uppsala for kind assistance in their old library.

Addresses of the authors

Jonas Ågren, Geodetic Research Division, National Land Survey, SE – 801 82 Gävle, Sweden, and Royal Institute of Technology (KTH), SE – 100 44 Stockholm, Sweden; jonas.agren@lm.se

Martin Ekman, Summer Institute for Historical Geophysics, Äppelträdgården E, Haraldsby, AX – 22 410 Godby, Åland; martin.ekman@ha.ax

ISSN 1798-1883 (print)
ISSN 1798-1891 (online)