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# An Investigation of the First Determination of Heights of Nordic Lakes above Sea Level

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## 1. Introduction

In the years 1754 – 1755 the first extensive levelling in the Nordic area was performed. Its purpose was special: To find the heights of a number of lakes above sea level. The lakes were the ones distributed across the southern part of Sweden, between the Baltic Sea in the east and the Sea of Kattegat in the west, including the largest lake in the Nordic area, Lake Vänern.

The major reason behind this early height determination of lakes was the interest in creating a possible sailing route between the Baltic and the Kattegat through Sweden, thereby avoiding the Sound of Öresund between Denmark and Sweden. Avoiding Öresund was a primary interest not only in order to shorten the sailing distance but also to escape the Danish customs duties. Ever since 1429 all non-Danish ships passing Öresund were liable to a heavy Danish customs duty. After the Danish provinces east of Öresund had become Swedish and Sweden had become a European great power, Sweden in 1645 managed to get exemption from the duty, but when Sweden lost its power status it also lost its exemption in 1720. So, when the lake levellings were made in the 1750s the Öresund duty was again in force; it would be so until 1857 when it was abolished according to an international treaty.

An additional reason behind the determination of lake heights was a growing interest in understanding the flow of water in nature. What water ended up in the Baltic Sea and what ended up in the Kattegat – and how did it get there?

At this time the topographical and hydrographical conditions in the Nordic area were quite unknown. The purpose of the present publication is to investigate this pioneering levelling of lakes, trying to estimate the errors in the obtained lake heights above sea level. This will involve handling the effects of postglacial land uplift as well as natural and artificial lake lowering.

## 2. The levelling of the lakes

The lake levellings, ordered by the King, were organized by the Royal Survey Office. They were carried out and reported by Collin (1755). The work was performed in two parts. The first part was made in 1754, spanning between Lake Vänern in the west and the Baltic Sea, or rather Lake Mälaren close the sea, in the east. The second part was made in 1755, again spanning between Lake Vänern in the west and the Baltic Sea in the east, but this time further south (Bay of Bråviken). In this way, using the Baltic Sea, the whole project constitutes a large levelling loop. (It may be mentioned here that some years earlier, in 1748, a levelling also had been performed along the water-falls in the river Göta älv between Lake Vänern and the Sea of Kattegat, but that was an incomplete levelling not covering the whole river.)

The levelling instrument used (and probably also the levelling rods) was constructed by the internationally renowned instrument maker Ekström and, therefore, most likely of high quality. Throughout, all lakes were considered horizontal surfaces. Accordingly, no levellings were performed along the lakes themselves, only between the lakes. Since several of the lakes are quite large this facilitated the work considerably. One must bear in mind that at this time there were hardly any suitable roads along which accurate levellings could be performed. A few examples of how a levelling between water surfaces at different heights could be arranged were given in Lefebvre (1753), where also the effect of the Earth's curvature was discussed.

In all more than 30 lakes were levelled. No details are known concerning the levelling routes between them, nor are any details known concerning the observation procedure. What we have today is the list of the resultant heights of the lakes. This knowledge later contributed to the first map indicating the direction of water flow between lakes, by Marelius (1773); see Figure 1.

## 3. Preliminary comparisons of the results with modern heights

Let us now start investigating this early levelling of lakes. For the investigation 17 main lakes are selected, reasonably distributed and including all large lakes. These lakes are listed in Table 1 as well as subsequent tables. Around the divide between the water-sheds of the Baltic and the Kattegat also small lakes quite close to each other were levelled; these have not been included. (Some of them could not even be identified.) The lakes in the tables are ordered geographically from the Baltic Sea westwards along the southern part of the levelling, to Lake Vänern, and then eastwards along the northern part back to the Baltic. For one of the lakes the name has shifted: The highest and most remote of the lakes had no name during the time of the levelling; it was simply named after the river flowing through it, Svartälven. Later it was called Karlsdalssjön but is nowadays known as Malmlången. In the tables the present name is used for this lake.

In Table 1 is given for each lake its measured height in Swedish feet, from Collin (1755), and in metres, obtained through 1 Swedish foot = 0.2969 m. The last row in the table showing values with asterisk needs some explanation. The actual levelling included Lake Mälaren but not the small difference between this lake and the Baltic Sea at the Stockholm sluice. This difference started to be recorded when a new sluice was built in 1755, the same year as the levelling was published. We have here used the average height difference

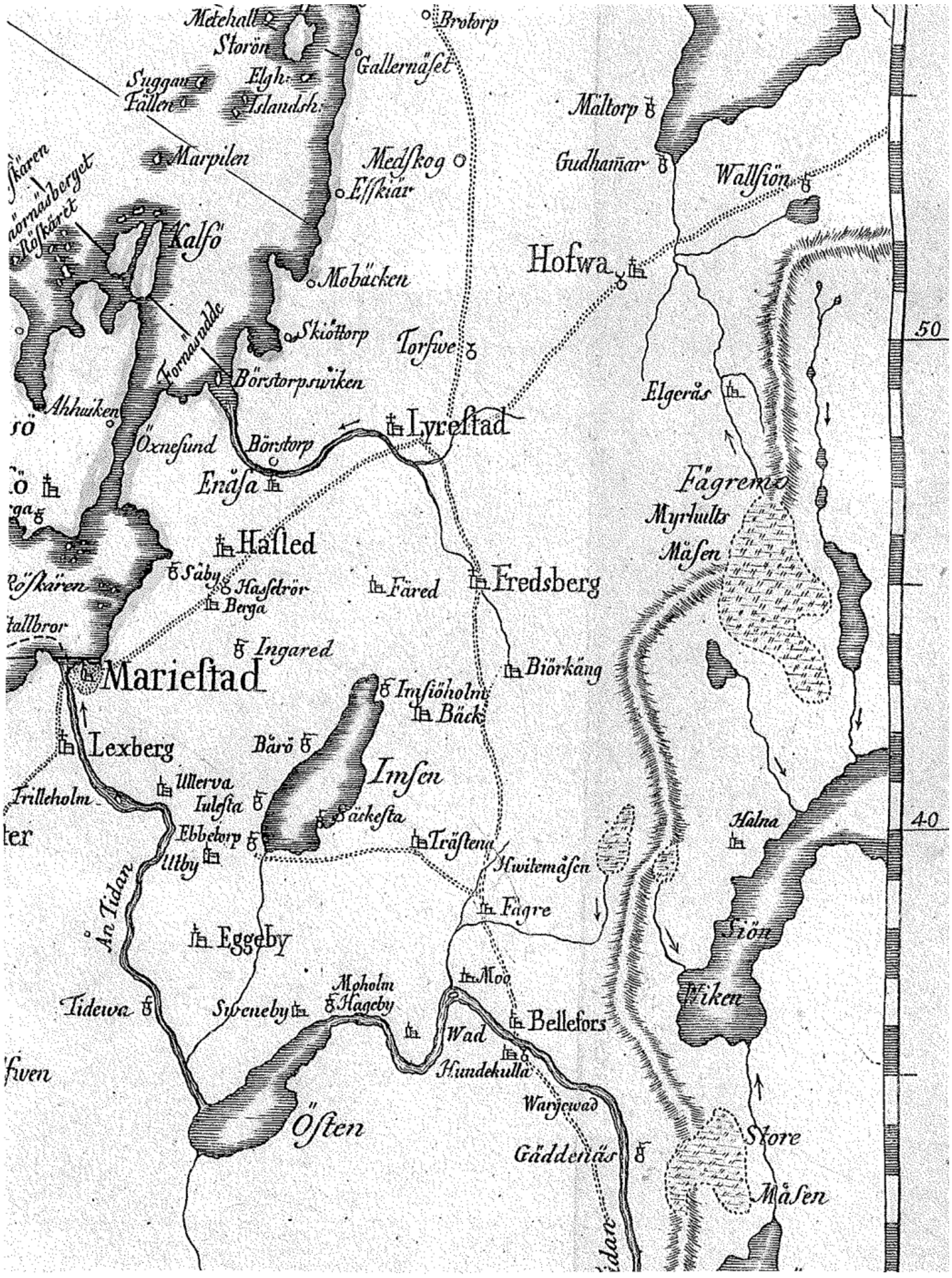


*Figure 1.* Part of the map of Marelius (1773) showing the direction of water flow between lakes east of Lake Vänern as well as the divide there between the water-sheds of the Kattegat and the Baltic. Several of the lakes had been included in the levelling of 1754 - 1755.

*Table 1.* Heights of the levelling of lakes in 1754 – 1755 (in Swedish feet and in metres), modern heights (in metres, height system RH 1900), and preliminary errors (in metres). Lakes are ordered from east (Baltic Sea) to west (Lake Vänern) and back to east (Baltic Sea) further north. Values with asterisk are explained in text.

| Lake | Meas. height (ft/m) | | Mod. height (m) | Prel. error (m) |
|---|---|---|---|---|
| Baltic Sea | 0.0 | 0.0 | 0.0 | - |
| Glan | 70.2 | 20.8 | 21.5 | - 0.7 |
| Roxen | 110.5 | 32.8 | 33.3 | - 0.5 |
| Boren | 242.2 | 71.9 | 73.3 | - 1.4 |
| Vättern | 292.7 | 86.9 | 88.5 | - 1.6 |
| Viken | 304.4 | 90.4 | 91.8 | - 1.4 |
| Unden | 393.7 | 116.9 | 116.8 | + 0.1 |
| Ymsen | 239.5 | 71.1 | 71.3 | - 0.2 |
| Vänern | 147.8 | 43.9 | 44.4 | - 0.5 |
| Skagern | 232.1 | 68.9 | 68.2 | + 0.7 |
| Toften | 252.7 | 75.0 | 74.8 | + 0.2 |
| Möckeln | 302.9 | 89.9 | 88.8 | + 1.1 |
| Malmlången | 502.1 | 149.1 | 147.1 | + 2.0 |
| Vikern | 380.0 | 112.8 | 112.7 | + 0.1 |
| Norasjön | 278.1 | 82.6 | 83.1 | - 0.5 |
| Väringen | 109.8 | 32.6 | 31.7 | + 0.9 |
| Hjälmaren | 79.8 | 23.7 | 21.9 | + 1.8 |
| Mälaren | 6.4 | 1.9 | 0.3 | + 1.6 |
| Baltic Sea | 4.7* | 1.4* | 0.0 | + 1.4 |

between Lake Mälaren and the Baltic Sea in the 1700s as calculated from the Stockholm sluice data; see Ekman (2011).

In Table 1 is further given for each lake its "modern" height according to the national Swedish height system RH 1900 (RH 00), the height referring to the mean level of the lake. All lakes have been identified on official topographic maps using the height system mentioned (up to 1970), and their heights taken from there. In case of the largest lake, Vänern, the mean height is taken from the long series of lake levels analysed by Wallén (1910), before the lake was artificially regulated. (There are two later height systems available but they are of no additional use for our purpose, rather they would make things more complicated.)

Taking the difference between the measured and the modern height we get a preliminary value of the error in the measured lake height. This is shown in the last column of Table 1. We may immediately notice two things. First there is a loop misclosure of 1.4 m, the sea level of the Baltic being 0.0 m at the top of the table but 1.4 m at the bottom. Second, the errors along the levelling are nowhere considerably larger than the loop misclosure. To continue our investigation we now need to discuss more closely the modern heights and how they can be made as comparable as possible with the measured ones.

## 4. Land uplift, lake lowering and renewed comparisons

The height system RH 1900 has a zero level defined as mean sea level in the Baltic Sea at Stockholm in 1900 (calculated from the long series of sea level observations at the Stockholm sluice). The levellings of Collin (1755) are based on a zero level coinciding with the sea level in the Baltic Sea at the Bay of Bråviken somewhat to the south of Stockholm in 1755. The difference in location between the zero definitions can be ignored here; the discrepancy in sea surface height due to differing salinity between the locations is insignificant (Ekman & Mäkinen, 1996). The difference in time between the zero definitions does not matter either; sea level itself did hardly rise or fall during these centuries (Ekman, 2009). Neither is there any reason to suspect a considerable temporary deviation of sea level here in the summer half of the year. Thus the zero levels can, for our purposes, be treated as identical.

What might have changed, however, are the heights of the lakes above the zero level. There are two processes involved here: postglacial land uplift on one hand and natural or artificial lake lowering on the other hand. Let us start with the land uplift.

The land uplift means that all lake bottoms have risen between 1755 and 1900 (or in some sense 1892, being the approximate epoch of the levellings governing the system RH 1900). Does this imply that the lake surfaces have risen by the same amount? In most cases: yes. There might, however, be exceptions; we will comment on that further below.

The amount of land uplift during the time span between 1755 and 1900 can be calculated from the uplift data of Ekman (1996, 2009) or Ågren & Svensson (2007). For the northern part of the levellings we adopt an (absolute) uplift rate of 5 mm/yr, for the southern part a rate of 4 mm/yr. This results in an uplift of the lakes along the northern part of 0.7 m and along the southern

*Table 2.* Heights of the levelling of lakes in 1754 – 1755, reduced modern heights, and errors (all in metres), after reduction for land uplift and lake lowering. Lakes are ordered as in Table 1.

| Lake | Meas. height (m) | Red. mod. height (m) | Error (m) |
|---|---|---|---|
| Baltic Sea | 0.0 | 0.0 | - |
| Glan | 20.8 | 20.9 | - 0.1 |
| Roxen | 32.8 | 32.7 | + 0.1 |
| Boren | 71.9 | 72.7 | - 0.8 |
| Vättern | 86.9 | 87.9 | - 1.0 |
| Viken | 90.4 | 91.2 | - 0.8 |
| Unden | 116.9 | 116.2 | + 0.7 |
| Ymsen | 71.1 | 70.7 | + 0.4 |
| Vänern | 43.9 | 43.8 | + 0.1 |
| Skagern | 68.9 | 67.5 | + 1.4 |
| Toften | 75.0 | 74.1 | + 0.9 |
| Möckeln | 89.9 | 88.1 | + 1.8 |
| Malmlången | 149.1 | 146.4 | + 2.7 |
| Vikern | 112.8 | 112.0 | + 0.8 |
| Norasjön | 82.6 | 82.4 | + 0.2 |
| Väringen | 32.6 | 31.0 | + 1.6 |
| Hjälmaren | 23.7 | 22.6 | + 1.1 |
| Mälaren | 1.9 | 0.5 | + 1.4 |
| Baltic Sea | 1.4* | 0.0 | + 1.4 |

part of 0.6 m during the time span in question. Thus all modern heights have to be reduced by these amounts to be comparable with the measured heights of 1755. This land uplift reduction has been made in Table 2, where the second column shows reduced modern heights.

As mentioned above, a lake surface does not necessarily have to rise by the same amount as its lake bottom. There might be special cases where the outlet of the lake, due to the uplift, is continually eroded. In such a case the lake surface is lowered relative to the ground, resulting in the lake level being more or less unaltered relative to the sea level. Such a case is the large lake of Mälaren close to the Baltic. In spite of the land uplift this lake level does not rise relative to the sea. Therefore, the modern height of Lake Mälaren has not been reduced for the land uplift as the other lakes. Its height has been reduced to 1755 by using the Stockholm sluice data as in Ekman (2011); see also Lilienberg (1891).

There might also be man-made processes affecting the long-term level of a lake. Towards the end of the 1800s one tried to artificially lower certain lakes to acquire more arable land. Such a case is Lake Hjälmaren to the west of Mälaren. Lake Hjälmaren was artificially lowered around 1885 by nearly 2 m. Therefore, also this lake has been treated as a special case here. For Lake Hjälmaren its height before the lowering according to data in Laurell (1885) has been used, and then reduced for land uplift to 1755.

Finally it should be mentioned that lakes also have become artificially regulated to avoid extreme levels, high and low, without considerably altering the mean level. Such a case is the largest of the lakes, Vänern, which is regulated in such a way since 1936. As mentioned the mean level of Lake Vänern is, therefore, taken from data in Wallén (1910), and then reduced for land uplift to 1755.

Now, taking the difference between the measured and the reduced modern heights in Table 2 we get values of the actual errors in the measured lake heights. This is shown in the last column of Table 2. As in Table 1 we recognize the loop misclosure of 1.4 m. What appears more clearly, however, is the distribution of the errors along the levelling. For the southern part the errors vary within 1 m around an average close to 0 (or more specifically around - 0.2 m). For the northern part the errors vary within approximately the same range, but around an average of + 1.3 m. This requires some further investigations.

## 5. Loop misclosure and a possible lake level change

We start by noting that the loop misclosure of 1.4 m is almost identical, within 0.1 m, to the systematic difference between the southern and the northern parts of the levelling loop, revealed by Table 2. This may be a coincidence due to random levelling errors happening to accumulate in that specific way. However, there might very well be a physical explanation rather than a statistical one to the two almost identical figures.

As stated earlier, the northern part of the levelling was made in 1754 and the southern part in 1755. It appears from Collin (1755) that the lake forming the link between the two parts is, as would be expected, the westernmost lake, Vänern, or in some unspecified way both Vänern and its close neighbour Skagern. A change in the level of Lake Vänern of 1.4 m between the levellings of 1754 and 1755 might then explain the systematic difference between the two parts as well as the misclosure. (Such a change might not have been noted if the levellings were connected to the lake at different locations.) Is this a realistic change in the lake level between two consecutive years?

The level of Lake Vänern has been observed systematically since 1807. This lake level series of more than 100 years was studied by Wallén (1910). His study reveals that Vänern shows considerable changes in level spanning over one or several years, without having any clear connection to seasonal changes in climate. Smaller lakes are usually known to have a strong seasonal variation in level caused by the annual melting away of snow. Vänern, however, being the largest lake in the Nordic countries, shows a completely different pattern because of its capacity to store very large amounts of water.

According to Wallén (1910) the total maximum difference in the level of Lake Vänern during the 100-year-period is 2.6 m. The maximum difference in lake level within one year amounts to 2.3 m, the level rising that year by 2.2 m in 8 months. There are also several years with the level rising or falling by about 1.5 m. Consequently, a change in the lake level of 1.4 m between 1754 and 1755 would be fully realistic.

A possible explanation of the misclosure of 1.4 m being equal to the systematic difference between the northern and southern parts of the levelling could thus be the following. In 1754 when the western end lake, Lake Vänern, was levelled for the northern part this lake was below normal. In 1755 when the levelling was resumed there (at another location) for the southern part the lake was 1.4 m higher and now above normal. Such a rise in lake level between the two years would cause a corresponding "jump" in the resultant heights, yielding systematically exaggerated heights along the northern part of the levelling as well as a loop misclosure of the same amount. In reality, of course, the misclosure might be explained by a lake level rise of less than 1.4 m combined with an accumulation of random errors.

In Table 3 we have recalculated the measured heights and the errors by eliminating the misclosure through assuming it to depend on a change in the level of the western end lake between the levelling years. The resultant errors now appear quite random. A crude estimate of the uncertainty in the lake heights through a formal calculation of the standard error, in spite of the heights being not quite independent of each other, yields

$$\sigma_H = 0.7 \text{ m}$$

*Table 3.* Recalculated heights of the levelling of lakes in 1754 – 1755, reduced modern heights, and recalculated errors (in metres), after elimination of misclosure according to text. Lakes are ordered as in Tables 1 and 2.

| Lake | Recalc. height (m) | Red. mod. height (m) | Recalc. error (m) |
|---|---|---|---|
| Baltic Sea | 0.0 | 0.0 | - |
| Glan | 20.8 | 20.9 | - 0.1 |
| Roxen | 32.8 | 32.7 | + 0.1 |
| Boren | 71.9 | 72.7 | - 0.8 |
| Vättern | 86.9 | 87.9 | - 1.0 |
| Viken | 90.4 | 91.2 | - 0.8 |
| Unden | 116.9 | 116.2 | + 0.7 |
| Ymsen | 71.1 | 70.7 | + 0.4 |
| Vänern | 43.9 | 43.8 | + 0.1 |
| Skagern | 67.5 | 67.5 | 0.0 |
| Toften | 73.6 | 74.1 | - 0.5 |
| Möckeln | 88.5 | 88.1 | + 0.4 |
| Malmlången | 147.7 | 146.4 | + 1.3 |
| Vikern | 111.4 | 112.0 | - 0.6 |
| Norasjön | 81.2 | 82.4 | - 1.2 |
| Väringen | 31.2 | 31.0 | + 0.2 |
| Hjälmaren | 22.3 | 22.6 | - 0.3 |
| Mälaren | 0.5 | 0.5 | 0.0 |
| Baltic Sea | 0.0 | 0.0 | - |

Superimposed on this is then a systematic effect making the northern lakes 1.4 m too high.

Collin (1755) suspects that his resultant heights may be systematically somewhat too low because of having not levelled along waters appearing to be horizontal. If such assumed horizontal waters in reality are not quite horizontal, heights will be systematically underestimated. Interestingly enough, our results in Table 3 (or 2) give no support to his suspicion. Rather they indicate a careful levelling under the circumstances given.

It was mentioned above that lakes smaller than the largest ones are usually known to have a strong seasonal variation caused by the annual melting away of snow in spring. At first glance this phenomenon might also be suspected to cause some systematic effect on the levelling of the lakes. However, such an effect has most probably been automatically avoided by the levellings simply being too difficult to perform too early in the year, before all snow and all melt-water had disappeared.

## 6. Conclusions

We have investigated the first determination of heights of Nordic lakes above sea level, taking into account the effects of land uplift and lake lowering since the levellings were made in the 1750s. We have found a loop misclosure of 1.4 m which is almost equal to a systematic difference found between the northern and the southern parts of the levelling. A possible explanation of this, wholly or partly, could be a rise of the connecting Lake Vänern between the two different years the two parts were levelled, a phenomenon that may not have been well known at the time. Eliminating the misclosure in such a way yields a random uncertainty of about 0.7 m in the lake heights.

Moreover, our results have been used to test a suspicion by the leveller that his published heights may be systematically somewhat too low because of not levelling along waters appearing to be horizontal. Interestingly enough our results give no support to his suspicion. Thus the levellings in this respect are shown to be better than assumed by the observer himself, a rather unusual discrepancy.

As stated in the Introduction, the major reason behind this early height determination of lakes was the interest in creating a possible sailing route between the Baltic and the Kattegat through Sweden. Based on the levelling between the lakes a route along the southern part of the levelling was recommended, whereas the northern part was rejected. Nothing more happened at that time, but half a century later the now well-known Göta Canal was built more or less along the recommended route. It required 58 sluices with lock gates to overcome the height differences.

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